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A MOLECULAR DYNAMICS MODEL OF MELTING AND GLASS TRANSITION IN AN IDEALIZED TWO-DIMENSIONAL MATERIAL I

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In a preparatory study of structural relaxations and plastic flow in a two-dimensional idealized atomic glass, the process of melting and quenching through a glass transition has been studied by computer simulation using a molecular dynamics model. In this model, the transition from a solid to a melt was observed to take place when liquid-like structural elements composed of dipoles of five- and seven-sided Voronoi polygons percolate through the two-dimensional structure of distorted hexagons in the form of strings. Such dipoles constitute discrete elements of excess free volume within which liquid like behaviour is established in the sense of reduced cohesion or local elastic moduli. Upon quenching the melt, the percolation condition

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of liquid-like regions is retained for under-cooled melts between the melting point and a glass transition temperature below which the percolation condition is broken and the thermal expansion is sharply reduced. The simulation that has used empirical pair potentials characteristic of Cu and Zr has substantially underpredicted the melting and glass transition temperatures and overpredicted the thermal expansion of $\text{Cu}_x\text{Zr}_{1-x}$ type glasses. These defects of the model can be partly attributed to the two-dimensional nature of the material, which stores larger concentrations of free volume than a corresponding three-dimensional material. In spite of these quantitative shortcomings, the model gives valuable insight into the topological features of the local atomic configurations at melting and upon vitrification.

0. NOMENCLATURE

Dimensional parameters

| | |
|----------------------------------|---|
| h_i | enthalpy of atom i |
| k | Boltzmann's constant |
| m_0 | mass per atom |
| p_i | pressure at atomic site i |
| r_{ij} | distance between atoms i and j |
| r_0 | fundamental length scale in pair potential |
| S_i | entropy of atom i |
| u_α | displacement parallel to cartesian axis α |
| u_β | displacement parallel to cartesian axis β |
| v_i | velocity of atom i |
| x, y | cartesian axes parallel to edges of undistorted simulation cell |
| $C(i)_{\alpha\beta\gamma\delta}$ | elastic constant defined at atomic site i |
| E_0 | energy scaling factor, binding energy in pair potential |
| H | average enthalpy per atom |
| V | average potential energy per atom |
| $S(i)_{\alpha\beta\gamma\delta}$ | elastic compliance defined at atomic site i |
| T | absolute temperature |
| $\alpha, \beta, \gamma, \delta$ | cartesian axes in the plane of simulation usually referring to local atomic sites |
| $\gamma(i)_{xy}$ | total tangential shear strain at atomic site i |
| $\epsilon(i)_{xy}$ | total tensor strain at atomic site i |
| ρ | density |
| $\sigma(i)_{xy}$ | tensor stress component at atomic site i |
| τ_i | maximum shear stress (Mohr circle) at atomic site i |
| $\Phi(r_{ij})$ | energy potential between atom pairs i and j |
| Ω_0 | volume (Voronoi polygon area) per atom in reference crystal |
| Ω | volume (Voronoi polygon area) per atom in disordered solid |

Derived units

| | |
|--------|------------------------------------|
| m_0 | unit of mass |
| r_0 | unit of length |
| E_0 | unit of energy |
| ϕ | unit of force, E_0/r_0 |
| v | unit of velocity, $\sqrt{E_0/m_0}$ |

| | |
|----------|--|
| π | unit of two-dimensional pressure or stress, E_0/r_0^2 (generalized to E_0/r_0^3 in three dimensions) |
| ρ_0 | unit of density, m/Ω_0 |
| θ | unit of temperature, E_0/k |
| η | unit of time, $r_0\sqrt{(m_0/E_0)}$ |

Normalized quantities

| | |
|-----------------|--|
| p^* | dimensionless pressure, p/π |
| v^* | dimensionless velocity, v/v |
| E^* | dimensionless energy, Φ/E_0 |
| F^* | dimensionless force, F/ϕ |
| T^* | dimensionless temperature, T/θ |
| ρ^* | dimensionless density, ρ/ρ_0 |
| σ_{xy}^* | dimensionless tensor stress element, σ_{xy}/π |
| τ^* | dimensionless maximum shear stress, τ/π |
| Ω^* | dimensionless atomic volume, Ω/Ω_0 |

1. INTRODUCTION

The modes of inelastic deformation in amorphous solids on the atomic scale below their glass transition temperatures, i.e. in their glassy behaviour range, and how these are related to the structure of these solids, has been of fundamental interest. The approaches taken to elucidate these phenomena in amorphous metals, polymers and network glasses have differed considerably along the traditionally different views taken by investigators in these fields. Nevertheless, these approaches have in general divided into two operationally different categories. In the first category are those studies that are based on kinematically acceptable, strain producing *ad hoc* local mechanisms, which involve cooperative atom or molecular segment motions, and which are constructed principally to model the inelastic shear resistance (Gilman 1968; Li 1986; Argon 1973; Robertson 1966; Bowden 1974; Spaepen 1977; Argon 1979; Argon & Kuo 1979; Argon & Shi 1983). In the second category are mostly studies based on detailed computer simulations of the structure of atomic (Kobayashi *et al.* 1980*a*; Maeda & Takeuchi 1982; see, for a review, Gaskell 1983) and polymeric (Theodorou & Suter 1986*a, b*) glasses, including some further analysis to explore the mechanisms of inelastic deformation in these simulated systems (Kobayashi *et al.* 1980*b*; Maeda & Takeuchi 1981; Srolovitz *et al.* 1983). Related to the latter are studies (Argon & Kuo 1979; Shi & Argon 1982; Argon & Shi 1982) using analogue models of two-dimensional atomic glasses based on the classical Bragg bubble model (Bragg & Lomer 1949). The various *ad hoc* models and computer simulations have been necessary because of the difficulty of obtaining information by direct experimental methods on the very local structural alterations that occur in the inelastic deformation in glasses. Some investigators have pursued a different line of approach relating inelastic deformations in glasses to extensions of well defined mobile crystal defects, such as vacancies and dislocations (for a review see Li 1986). In our view, however, these have not furnished many quantitatively useful explanations of either the topological features of the deformation process or its kinetics.

In this and the following three papers (Deng *et al.* 1989*a* (II), *b* (III), *c* (IV)), we report the

results of a series of computer simulations of the structure, its phase transitions, structural relaxations and plastic deformation under stress of a two-dimensional model atomic glass. In these simulations, our goal was not to obtain quantitatively exact descriptions of structures, of physical processes of phase change, or of mechanical properties, results that can be compared directly with experimental data on metallic glasses. There are certain topological limitations of a two-dimensional simulation that does not permit this. Rather, our goal has been to elucidate complex topological processes of a cooperative nature, and to develop qualitative understanding and semi-quantitative scaling laws for them.

One principal object of the study was the simulation of structural ageing and the subsequent cooperative atom motions occurring in local shear transformations upon large strain shearing. To obtain the initial structures for these studies in glassy assemblies in a non-arbitrary way, it was decided to obtain all such initial states by melting and quenching the two-dimensional atom assemblies. In this communication, we discuss only our findings related to melting and the glass transition upon quenching.

In several instances we have encountered significant constraints on the kinematics and the distributed nature of the processes of shear relaxation even in relatively large two-dimensional periodic cells, indicating the wisdom of preferring a large two-dimensional array of a fixed number of atoms, rather than distributing them in a more confined but three-dimensional periodic cell – when the overall number could not be arbitrarily increased for reasons of tractability in the simulation – not to mention the difficulty in visualization of the local transformations.

2. DETAILS OF THE SIMULATION

2.1. *The atomic potentials*

The simulations reported here and in parts II–IV make use of standard molecular dynamics (MD) techniques (Allen & Tildesley 1987). In (MD) Newton's equations of motion for a two-dimensional system of N particles are integrated using a fifth-order predictor–corrector algorithm. Periodic boundary conditions are imposed on the simulation cell, and through a lagrangian formulation (Andersen 1980) the components of the basis vectors defining the cell are allowed to vary in accordance to any imbalance between the internal stress and an externally prescribed stress. The system temperature is defined in terms of the average kinetic energy of a particle. To maintain the system at a particular temperature the particle velocities are rescaled at every time step of the simulation. If the system has reached thermal equilibrium then the instantaneous temperature, without velocity rescaling, will fluctuate about a mean value, which does not change in time.

The atomic interactions in the present simulations are given by pair potentials of the Lennard-Jones type (Kobayashi *et al.* 1980*a*):

$$\Phi(r_{ij}) = E_0[(r_0/r_{ij})^m - (r_0/r_{ij})^n] - a'r_{ij} + b', \quad (1)$$

where $\Phi(r_{ij})$ is the potential energy of interaction between atoms i and j at a separation distance r_{ij} , r_0 and E_0 are length and energy parameters of the potential, and the constants a' and b' are chosen to obtain smooth truncation of the potential some place between the third and fourth nearest neighbours. The exponents m and n are phenomenologically assigned to obtain a desired form for the potential.

In our simulations two different forms of the potential were used. In the early phase of the study of melting and stability of the quenched glassy state of a single component solid a Lennard-Jones potential was used with $m = 12$, $n = 6$, $r_0 = 3.405 \text{ \AA}^\dagger$, and an energy to atomic mass ratio $E_0/m_0 = 2.485 \times 10^8 \text{ cm}^2 \text{ s}^{-2}$, to give a fundamental atomic period $\eta = r_0(m_0/E_0)^{\frac{1}{2}} = 2.16 \times 10^{-12} \text{ s}$ ($E_0 = 1.034 \times 10^{-2} \text{ eV}$, and $m_0 = 6.656 \times 10^{-23} \text{ g}$). These particular parameter values are appropriate to the noble gas solid argon. The simulations were carried out under two constant external pressures: $p^* = 0$, and 1, in units of $\pi = E_0/r_0^3$ ($= 41.9 \text{ MPa}$ in three dimensions), to explore the effect of pressure on structural stability. In a typical simulation run, the time-step size has been taken at 1% of the fundamental period η , which makes it $2.16 \times 10^{-14} \text{ s}$ in this case. As we discuss below briefly, this single-component solid crystallized exceedingly rapidly. Therefore, simulations with this system were not carried much further.

In the main simulation, which will be the subject of this and parts II–IV, a modified Lennard-Jones potential of the type introduced by Kobayashi *et al.* (1980a) for a two-component glass, such as $\text{Cu}_x\text{Zr}_{1-x}$, was used. For the system considered here, $x = 0.5$, $m = 8$, $n = 4$. The Cu and Zr atoms have been labelled as A and B respectively, and their characteristic parameters have been taken as follows: $r_0 = 2.556 \text{ \AA}$, $m_0 = 10.63 \times 10^{-23} \text{ g}$, $E_0 = 0.15 \text{ eV}$ for the Cu (A) atoms, giving a fundamental atomic period $\eta_A = 5.4 \times 10^{-13} \text{ s}$, a fundamental stress unit $\pi_A = 1.437 \text{ GPa}$ (in three dimensions). The interactions between the Zr atoms (B–B) and the Cu and Zr atoms (A–B) were obtained according to the following ratios: $r_{0BB}/r_0 = 1.244$; $r_{0AB}/r_0 = \frac{1}{2}(1 + r_{0BB}/r_0) = 1.122$; $E_{0BB}/E_0 = 1.94$; $E_{0AB}/E_0 = (E_{0BB}/E_0)^{\frac{1}{2}} = 1.393$. Furthermore, the two coefficients a' and b' were chosen such that interatomic forces vanish at a critical cut-off distance of $r_c = 2.5r_0$. The resulting pair potential for copper is shown in figure 1. The two other pair potentials look very similar. To enhance stability in this simulation, an external pressure of $p/\pi = 1.0$ was chosen (i.e. 1.437 GPa in three dimensions). This is a substantial pressure, and its effect on the quantitative aspects of the simulation must be kept in mind. The time step size was taken at 5% of the fundamental period, i.e. at $1.1 \times 10^{-14} \text{ s}$.

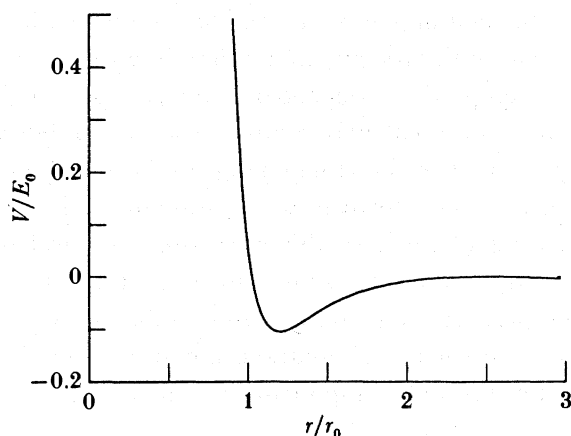


FIGURE 1. The interatomic pair potential between Cu atoms used in the simulation with the two-component (Cu–Zr) material.

$\dagger 1 \text{ \AA} = 10^{-10} \text{ m}$.

The method by which the simulation cell is allowed to change volume and shape is based on a lagrangian, which contains a kinetic energy term associated with the motions of the cell vectors (Andersen 1980; Parinello & Rahman 1981). A mass parameter appears in this term; although it can be assigned any value, a proper choice is important, as it governs how quickly the cell vectors respond to the imbalances between the prescribed external pressure and the fluctuating internal pressure. We discuss the effect of this so-called wall mass on the simulations and its proper choice in Appendix I of the accompanying paper on structural relaxations (part III). The magnitude of this mass in the present simulation was 4.

2.2. Simulation cell

The simulation cells that were chosen in this study contained 144 atoms initially arranged on a 12×12 grid in a hexagonal close packed pattern (see figure 9a). Because of the hexagonal symmetry, the simulation cell was 11.5% longer in the horizontal direction than in the vertical direction. Areas allocated to each atom were delineated by Voronoi polygons constructed in the conventional way. This was done under every circumstance, and proved to be of great value in visualization of distortions, but also in determination of strain increments at each atomic site and the level of coordination of atoms to their surroundings. To simulate properties of a two-dimensional solid of infinite extent in the plane of simulation, periodic boundary conditions were used throughout the simulations. In the simulations of the binary atom solids of $\text{Cu}_{0.5}\text{Zr}_{0.5}$, the Cu(A) and Zr(B) atoms were initially assigned randomly.

2.3. Melting and quenching simulations

The melting of the two-dimensional crystal lattice was achieved by step-wise increase of the system temperature. The dimensionless temperature $T^*(= T/\theta)$ was increased in steps that ranged from 0.025 to 0.1 (the normalized temperature $\theta = E_0/k$ is 120 K for the one-component solid and 1739 K for the two-component $\text{Cu}_{0.5}\text{Zr}_{0.5}$ solid). Between increments, the temperature was held constant by rescaling the particle velocities. To achieve equilibrium as defined by cumulative-averaged properties no longer changing with time, it was necessary to equilibrate for $(3-4) \times 10^3$ time steps at temperatures below the melting temperature and for $(3-5) \times 10^3$ time steps above the melting temperature. During the melting process, where large and discontinuous changes occur in the system, often more time steps were necessary to achieve stabilized melt structures. All temperature-dependent intensive properties of the equilibrated system for the heating runs have been calculated after structural stability was attained at the new temperature, judged by an absence of further change in the average volume per atom. The same was done on cooling in the melt, down to where time-dependent structural relaxations take progressively longer periods of time. In this case, the structural relaxations were only carried out partly for each cooling step to freeze in a structure typical of that of the subcooled melt. In this range, each temperature reduction of about 0.05–0.1 was followed by only 10^3 time steps of stabilization. Such quenching simulations amount to very high rates of cooling in the range of $3 \times 10^{11} \text{ K s}^{-1}$ for the one-component system and $3 \times 10^{10} \text{ K s}^{-1}$ for the two-component systems.

The atom coordinates for each state in the melting and quenching runs were stored for future use in relaxation studies.

3. RESULTS

3.1. *Temperature dependent changes in atomic volume, enthalpy and potential energy*

The changes in the average atomic volume (average Voronoi area) Ω^* , enthalpy per atom H^* , and potential energy per atom V^* , all in normalized units, resulting from the changes in system temperature have been determined for the melting and quenching runs for both the single-component solid under two pressures (0, 1.0), and the two-component solid under a pressure of 1.0, for each step of the runs.

The problem of relating the two-dimensional results to three-dimensional properties has been dealt with systematically as follows.

1. Stresses and pressures are defined for use in three dimensions, but operationally are handled in intermediate level computations in two dimensions, i.e. as forces per peripheral length. In final presentations, all results are normalized by the three-dimensional pressure unit (E_0/r_0^3) so the reader can determine three-dimensional properties if desired;

2. All intensive properties of interest obtained from the simulation have been presented per atom when given in dimensioned units. In all other cases they have been given in non-dimensional form, but still per atom.

Thus the two-dimensional simulation cell has been treated for three-dimensional purposes, as if it had a thickness of r_0 , although in every instance the simulation was constrained to lie in a plane.

Whereas the average Voronoi polygon area per atom Ω^* in units of the fundamental area r_0^2 is obtained directly from the simulation plane, the instantaneous potential energy per atom V^* and enthalpy per atom H^* , on a per atom basis were obtained as follows:

$$V^* = \frac{1}{nE_0} \sum_i^n \sum_{j \neq i}^n \frac{1}{2} \Phi(r_{ij}), \quad (2)$$

$$H^* = \frac{1}{nE_0} \sum_i^n \frac{1}{2} m_i v_i^2 + V^* + p\Omega^*. \quad (3)$$

The results for Ω^* , H^* , and V^* are plotted as a function of T^* in figures 2 and 3 at two external pressures p^* of 0 and 1.0 (in units of $\pi = E_0/r_0^2$) for the single-component solid, and figure 4 for the two-component solid (constructed of equal numbers of atoms A and B) at an external pressure of 1.0.

In all cases, the properties, Ω^* , H^* , and V^* increase monotonically with increasing temperature. The changes in the properties of the one-component solid (cases I and II), and the two-component solid (case III) with increasing temperature are summarized in table 1, where α_0 and α_m are the coefficients of linear thermal expansion at zero temperature and the melting point respectively. It can be seen that the ratio α_m/α_0 is 6–9 for the one component solid and 2.5 for the two-component solid. Across the melting point, α does not increase significantly. The same is true for the rate of change of enthalpy and potential energy per atom. This suggests that the melting observed here is more a process of topological transition, and manifestations of a discontinuous behaviour in the rates of change of intensive properties are weak. This conclusion will be reinforced later by inspection of the topological features associated with the melting process. The latter are to be expected because the limited system size and simulation durations make the observed transition necessarily smeared out.

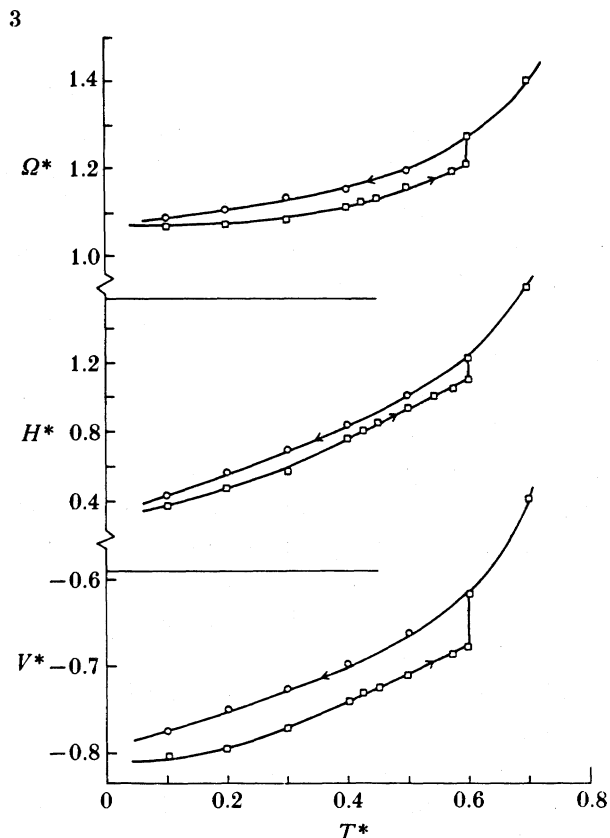
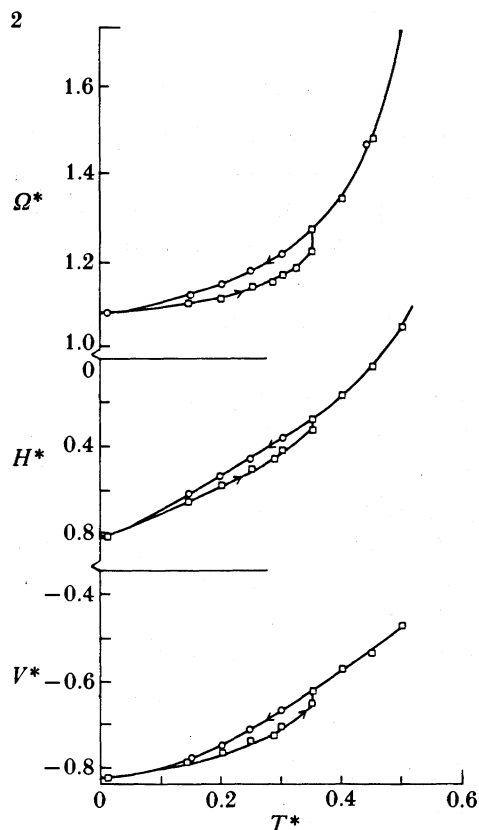


FIGURE 2. Temperature variations of dimensionless volume Ω^* , enthalpy H^* , and potential energy V^* , all on a per atom basis for the one-component model at external pressure of $p^* = 0$. Heating and cooling results are denoted by squares and circles respectively.

FIGURE 3. As figure 2, except $p^* = 1.0$.

The fractional linear expansion in these model solids upon melting range from about 2.14×10^{-2} for the one-component system to 5.49×10^{-3} for the two-component system, giving projected volumetric expansions on melting of 6.42×10^{-2} for the one-component solid and 1.65×10^{-2} for the two-component solid. These results do not compare too well with the known volume expansions of noble gas solids (12.7% for Ar) and close-packed metals (4.2% for Cu).

The absolute agreement in the melting points is also unsatisfactory. In the one-component solid, the melting point under no pressure is 42 K and under a pressure of $p^* = 1.0$ (42 MPa) is 72 K. Compared with the melting point of solid Argon of 84 K at atmospheric pressure, both values are too low. The same is true for the two-component solid, which, at a pressure of $p^* = 1.0$ (1.437 GPa), has a melting point of 434 K, whereas the melting point of the intermetallic compound $\text{Cu}_{0.5}\text{Zr}_{0.5}$, which it is to simulate, has a melting point of 1253 K at atmospheric pressure.

It is worthwhile to note here that the phase diagram of the two-dimensional Lennard-Jones system has been studied by Monte Carlo simulation (Barker *et al.* 1981). The triple point conditions are known to be $\rho_t^* = 0.815$, $T_t^* = 0.415$, and $p_t^* = 0.0056$. As p_t^* is essentially zero, we can compare the melting temperature T_m^* and corresponding density ρ_m^* in figure 2 with these values. We find that $T_m^* = 0.353$ and $\rho_m^* = 0.812$. We believe that the reason why our T_m^* value is lower than the literature value is because our range cut-off r_c is somewhat shorter,

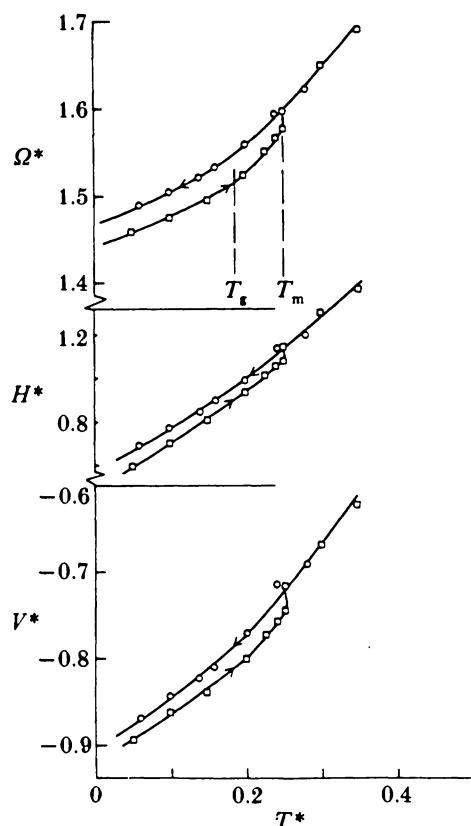


FIGURE 4. As figure 3, except the system is the two-component (Cu-Zr) model.

TABLE 1. MELTING AND SOLIDIFICATION OF TWO-DIMENSIONAL MODEL SOLIDS

| case | ρ^* | π/r_0 (GPa) | θ (K) | T_m^* | T_g^* | α_0 (K ⁻¹) | α_m (K ⁻¹) | $(\Delta\alpha/\alpha)_m$ | $(\Delta H/H)$ |
|------|----------|-----------------|--------------|---------|---------|-------------------------------|-------------------------------|---------------------------|----------------|
| 1 | 0 | 0.0419 | 120 | 0.35 | — | 6.62×10^{-4} | 4.10×10^{-3} | 2.14×10^{-2} | — |
| 2 | 1.0 | 0.0419 | 120 | 0.60 | — | 2.83×10^{-4} | 2.69×10^{-3} | 2.43×10^{-2} | 0.108 |
| 3 | 1.0 | 1.437 | 1739 | 0.25 | 0.18 | 7.35×10^{-5} | 1.85×10^{-4} | 5.49×10^{-3} | 0.057 |

and this effect is known to lower the melting point. In addition, we have not taken into account the long-range correction to the pressure or correction for system size dependence.

The pressure dependence of the melting point, as simulated in our computations for the one-component solid is 11.3 K/GPa. This is difficult to compare with actual experimental results, as the corresponding information is unavailable for noble gas solids. On a normalized basis, however, a comparison is possible where the result from the simulation $(\Delta T_m/T_m)/(\Delta P/\pi)$ is 0.526. The corresponding information for an alkali metal, such as Na is 0.338, where $\pi = (E_0/r_0^3)$ is taken as 0.65 GPa, as scaled down from the given ratio of Cu, on the basis of their different elastic properties. Clearly, the corresponding effect for a noble gas solid, such as argon, should be larger and more comparable with our simulation.

From the information discussed above, we conclude that our simulation in two dimensions underestimates the absolute melting points, but gives qualitatively acceptable results for the dependence of the melting point on pressure on a normalized basis. Furthermore, the results of the simulation on the linear coefficient of expansion for the two-component solid at

$7.35 \times 10^{-5} \text{ K}^{-1}$ at low temperatures is quite high when compared with what might be expected for $\text{Cu}_{0.5}\text{Zr}_{0.5}$, estimated to be about $1.1 \times 10^{-5} \text{ K}^{-1}$ as an average for Cu and Zr, at equal volume fractions. This behaviour of the simulation, however, is not unexpected, as two-dimensional models of disordered media are expected to give overestimates of specific volume and related properties, such as thermal expansion. As the purpose of the simulation of melting was principally to obtain non-arbitrary starting coordinates for atoms in a disordered medium in the solid state upon quenching from the molten state, we do not attach too much importance to the relatively poor agreement between the results of the simulation and the absolute melting behaviour. We will, however, find many points of interest in the topological features of the melt and its alterations upon quenching; we discuss these results below.

When the melt is cooled by systematically reducing the temperature in steps with intervening short periods of structural relaxation to achieve a condition of very slow change of properties (*ca.* 1000 time steps), the intensive properties decrease continuously and monotonically. Although the decreases above the melting temperature follow the same behaviour of the heating curve, in reverse, this is not so below the melting point, as figures 2–4 clearly show. At these cooling rates, which are in the range of 10^{11} K s^{-1} , the liquid phase has no time to crystallize by nucleation, but follows a temperature dependent contraction roughly parallel to that of the solid hexagonal close-packed phase, in an undercooled state, for the one- and two-component materials under pressure (figures 3 and 4). In the one-component material the slope of the cooling curve always remains larger than that of the corresponding crystal, making the two curves meet before reaching zero temperature. This indicates that the one-component system under no pressure in the undercooled state is unstable and crystallizes even at these very high rates of quenching. In comparison, both the one-component system under pressure, as well as the two-component system under pressure, do not join onto the hexagonal crystal and remain at least quasi-stable in the disordered form.

In real systems, when a changeover occurs in the rates of change of intensive properties in a very narrow range of temperature from a liquid-like behaviour to solid-like behaviour, without any discontinuity in the property itself, the material is said to undergo a glass transition. This is indicated fairly clearly in figure 4 for the two-component material, where a glass transition is found at a normalized temperature of 0.18, in comparison to the melting point at 0.25.

There exist several simulation studies of the Lennard-Jones system adopted here. Monte Carlo results have been reported on melting in two dimensions by Abraham (1982), and Tobochnik & Chester (1982), and on the glass transition in two dimensions in one and two component systems by Wang & Chester (1987). In three dimensions, molecular dynamics results on melting and on crystallization (Nose & Yonezawa 1985*a*), and glass transition (Fox & Andersen 1985; Nose & Yonezawa 1985*b*) are available.

3.2. Changes in topological features

3.2.1. Radial distribution function (RDF)

The changes in the RDF of the three materials that occur upon heating are quite similar. In figure 5*a–d* we show the typical sequence for the two-component solid. At increasing temperatures, the peaks occur at larger distances and they become broader and less intense.

Two important qualitative changes occur in the RDF near the melting point. First, the second and third peaks, which are clearly separated in the expanded hexagonal close-packed structure

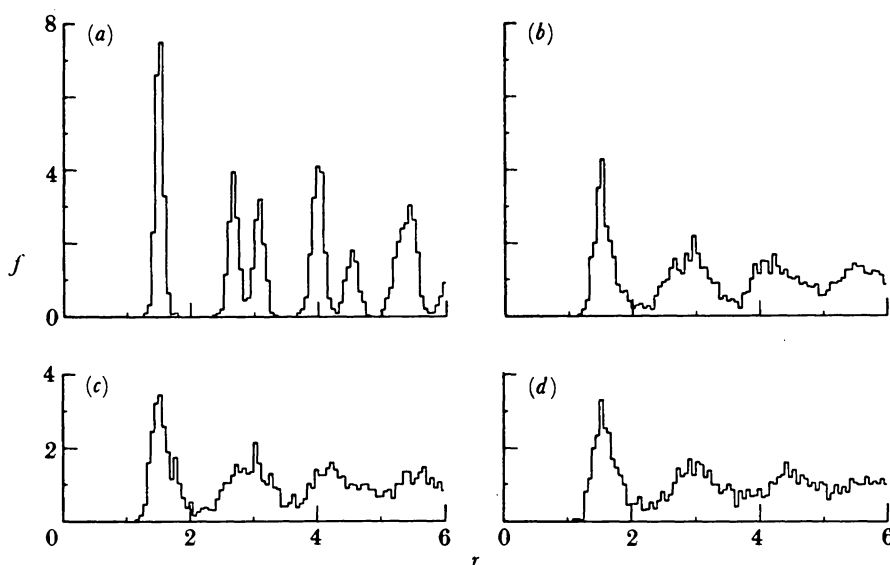


FIGURE 5. Temperature dependent alterations in the RDF in the two-component material upon heating at $p^* = 1.0$: (a) $T^* = 0.05$; (b) $T^* = 0.225$; (c) $T^* = 0.25$ ($T = T_m$); (d) $T^* = 0.3$.

below T_m , merge together and form a very broad and short peak at $T > T_m$. This also occurs for the fourth and fifth peaks, which also spread out and fuse together in the melting process. Second, the clear separation between peaks present below T_m begins to be filled up near the melting point. This is particularly evident in the larger separation between the first and second, and between the third and fourth peaks. Thus, above the melting point, the RDF has non-zero values at all separations and the previously well-defined first six peaks fuse into three broad and spread-out peaks. Some of these changes are also observable just below the melting point. This, as we have already pointed out, is a result of the limited system size and short durations of the simulation, which spreads out transitions.

The changes in the RDF at melting reflect the changes and smearing out of structural order. This is best followed from the ideal two-dimensional hexagonal lattice shown in figure 6 and its RDF reproduced immediately under it. Clearly, the second and third, and the fourth and fifth peaks are relatively close together, so that increased amplitude of thermal motion in the solid state and eventually the even larger amplitudes of atom motions possible in the molten state should smear out and fuse these peaks together.

The changes in the RDF upon quenching for the three materials are qualitatively different in the one-component solid from the two-component solid in the time range of the simulation. The one-component solid under no confining pressure crystallizes nearly completely, as figure 7*a–d* shows clearly. The same is also true under even a pressure of 1.0. In comparison, for the same quenching schedule, the two-component material under a pressure of 1.0 is much more stable and reaches a reasonably stable state at a temperature of 0.06, as figure 8*a–c* shows. Even here, however, as is seen in figure 8*c*, the second and third peaks have clearly begun to separate, and there are indications that the fifth peak is about to separate from the fourth peak. Nevertheless, the distribution is still continuous and without any breaks. We take this instantaneous state of well-relaxed structure as a borderline case between a defective crystal and a glass with considerable topological short-range order.

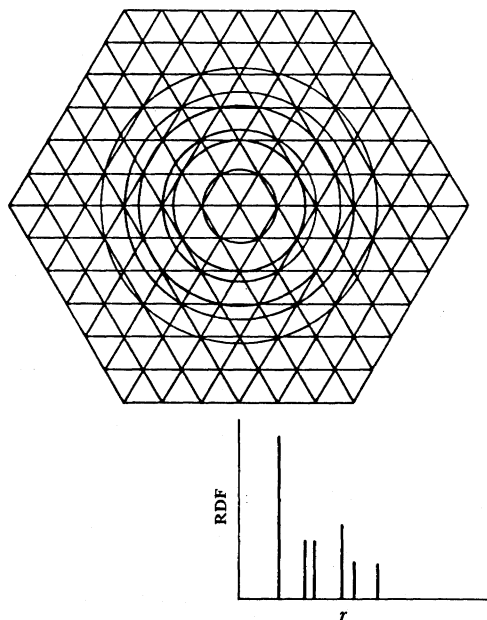
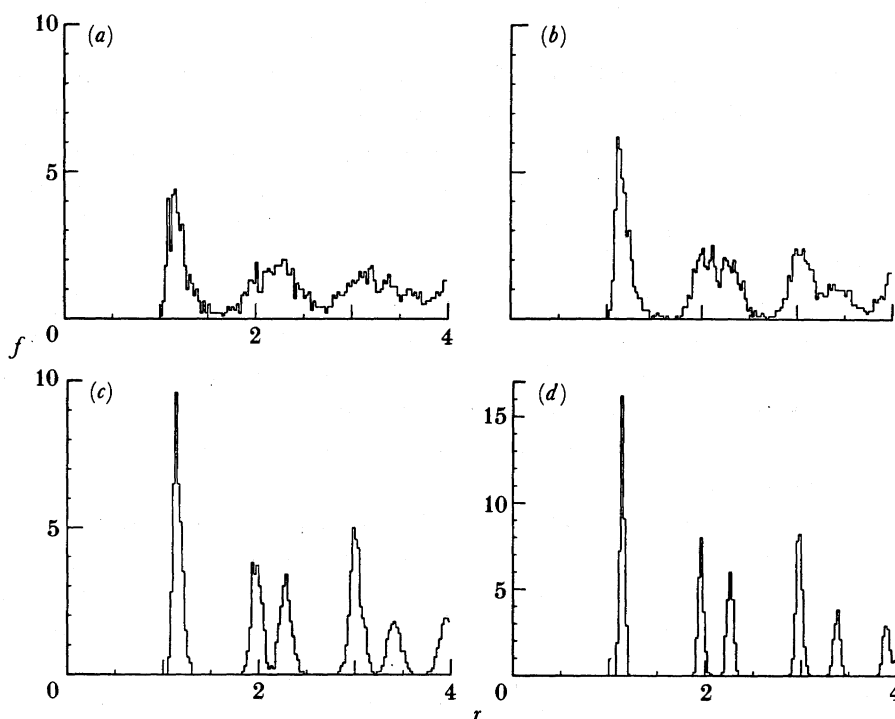


FIGURE 6. The RDF peak positions in a perfect hexagonal structure.

FIGURE 7. The temperature dependent alterations in the RDF in the one-component material upon quenching at $\rho^* = 0$: (a) $T^* = 0.35$; (b) $T^* = 0.25$ ($T = T_m$); (c) $T^* = 0.15$; (d) $T^* = 0.05$.

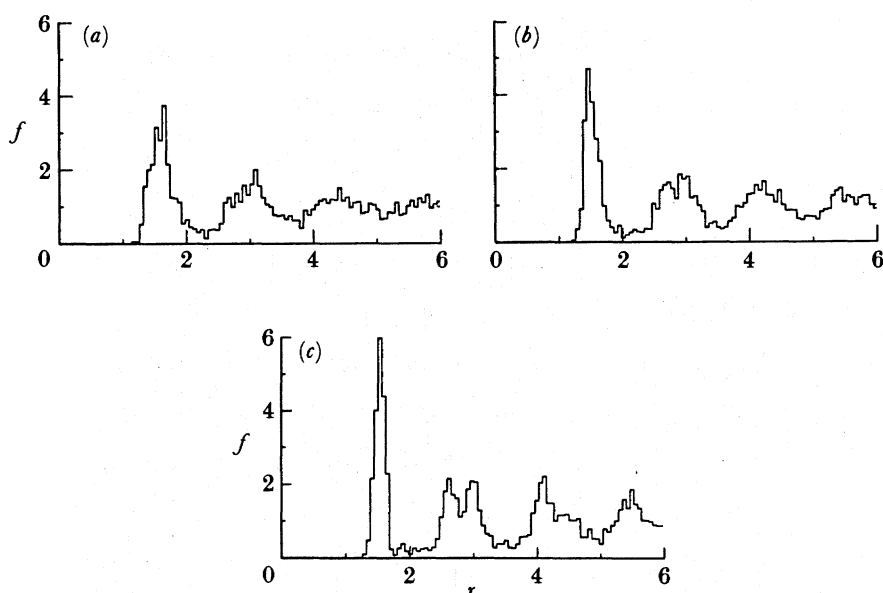


FIGURE 8. Temperature dependent alterations in the RDF in the two-component material below the melting point at an external pressure of $p^* = 1.0$: (a) $T^* = 0.24$; (b) $T^* = 0.14$; (c) $T^* = 0.06$.

3.2.2. Topological changes in melting

The topological changes occurring in melting for the one-component and two-component systems under a normalized pressure of unity ($p^* = 1$) are shown in figures 9 and 10. The areas per atom are delineated by Voronoi polygons. Their changing sizes, shapes, and associations reveal important details that cannot be perceived from the RDFs.

In the sequences in figure 9, the changes in the one-component material are shown with increasing temperature. It must be born in mind that while each picture reflects the structure in near equilibrium after $(1-4) \times 10^3$ time steps, they are nevertheless only a snapshot in time, showing only one of a large number of statistically equivalent configurations with the same overall energy. Figure 9a shows the simulation cell of 144 atoms, which still shows a relatively well-ordered hexagonal atomic arrangement of the solid at half its melting point ($T^* = 0.3$). While all polygons are still hexagons at this temperature, careful inspection of these shows that some of them have acquired thermal motion induced small distortions. At a normalized temperature of 0.5 ($T = 0.83 T_m$), all polygons are still hexagons, but many of them show quite large distortions. In figure 9b, at the melting point ($T_m^* = 0.6$) after only 1000 time steps before complete melting has occurred, qualitatively different topological changes are apparent. Now, most of the hexagons are quite visibly distorted, and many of the hexagons have been transformed into pentagons (5) and heptagons (7). Furthermore, accompanying the five- and seven-sided polygons, corners sharing four edges also appear as a transient form for the first time. What is more, the five- and seven-sided polygons show an affinity to each other. Figure 9c shows the structure at the melting point after 4000 time steps where structural equilibrium has been reached and no further expansion occurs. The simulation cell in the molten state has acquired a significant shear distortion without any applied shear stress. This can be traced to an artefact related to the smallness of the simulation cell. In a cell of amorphous material of only 144 atoms subjected to a periodic boundary condition, the repetition in space of the borders

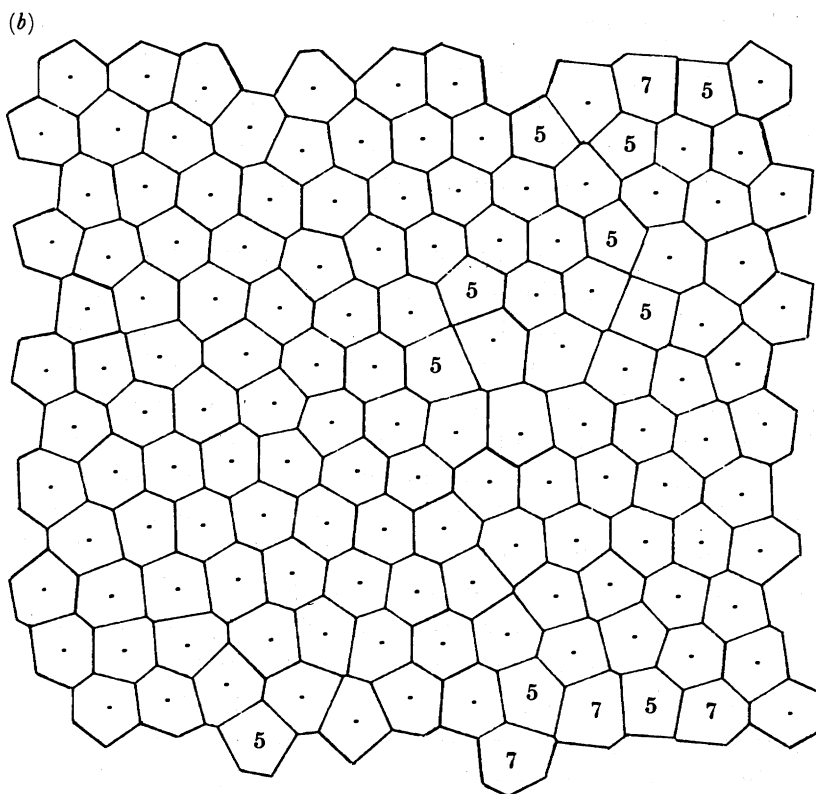
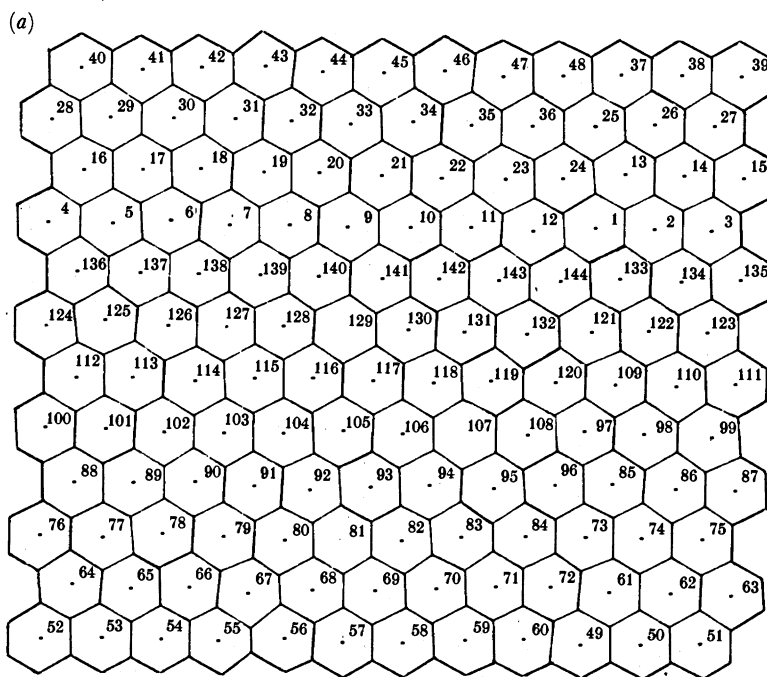


FIGURE 9*a, b*. For description see opposite.

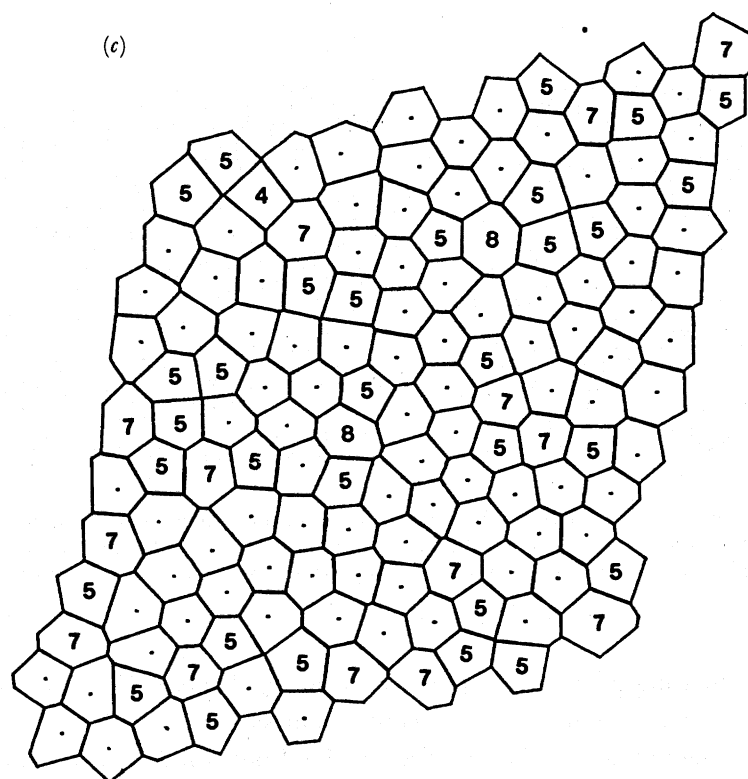


FIGURE 9. Temperature dependent alterations in the structure of the one-component material upon heating at an external pressure of $p^* = 1.0$: (a) $T^* = 0.3$ ($T = 0.5T_m$); (b) $T^* = 0.6$ ($T = T_m$); incomplete melting after only 10^3 time steps; (c) $T^* = 0.6$ ($T = T_m$), complete melting after 4000 time steps.

of the cell constitutes a paracrystal in which the periodic border material makes up a significant volume fraction. This volume fraction can increase even further if the cell is allowed to shear in the liquid state, where the internal resistances to deformation have become very small. We ignore this distortion. More importantly, in the simulation cell of the molten material, a large number of the Voronoi polygons have now become five- and seven-sided, indicated by the numbers in the polygons, and these have associated themselves in strings, in which four-edged corners also appear frequently. In most instances, the strings percolate through the cell, as is shown in the bottom of figure 9c. At still higher temperatures, the percolating string-like association of the five- and seven-sided polygons in the simulation cell become an even more prominent feature.

The corresponding distortions upon melting in the two-component material under a normalized pressure of unity ($p^* = 1$) are shown in the sequences in figure 10. In figure 10a, the initial hexagonal arrangement is shown at $T^* = 0.1$ ($T = 0.4T_m$). The Voronoi polygons containing the stiffer Zr atoms are identified by the small circles in the polygons. Because of the differences in size and local binding between the Cu and Zr atoms, many polygons appear already significantly distorted. Any apparent clustering of the Zr atoms in the simulation cell is illusory, because the Cu and Zr atoms were assigned to polygons purely randomly. At $T^* = 0.2$ ($T = 0.8T_m$), again the hexagons show similarly increased distortions, as in the one-component material. Figure 10b shows the high temperature melt at $T^* = 0.3$ ($T = 1.2T_m$) with percolating strings of five- and seven-sided polygons. At the same homologous temperature

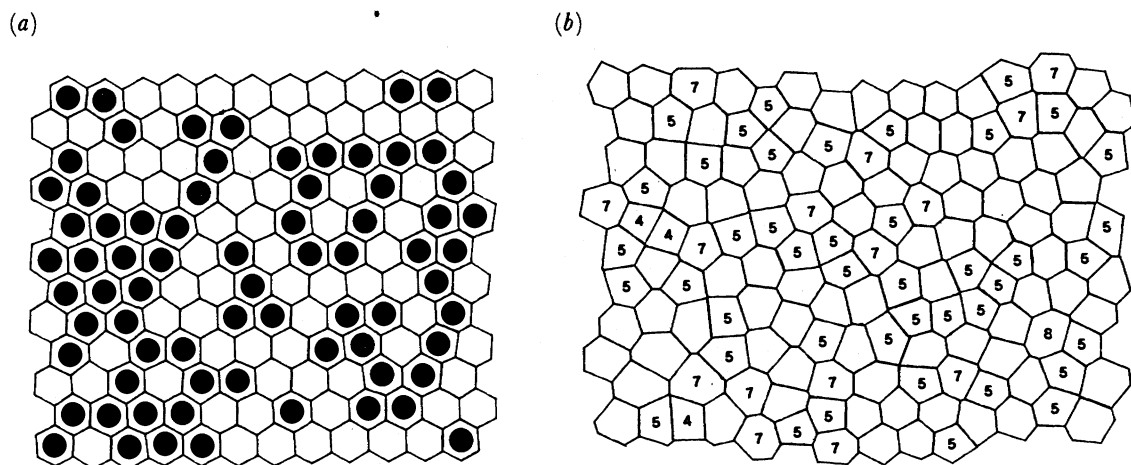


FIGURE 10. Temperature dependent alterations in the structure of the two-component material upon heating at an external pressure of $p^* = 1.0$: (a) $T^* = 0.1$, the dark atoms are Zr, the light ones Cu; (b) $T^* = 0.3$ ($T = 1.2T_m$) after complete melting the five- and seven-sided polygons associate into percolating strings that traverse across the cell.

with respect to the melting point, the departures from regularity of the hexagons are larger in the two-component system than in the one-component system.

We demonstrate below that the five- and seven-sided polygons usually associate themselves into a structural dipole of liquid-like material, and that the further percolating string-like associations of these dipoles imparts fluid-like behaviour to the structure.

3.2.3. Topological changes in undercooled melts and in a glass transition

The topological changes in the arrangement of atoms undergoing quenching is shown in figure 11 for the two-component system, which is typical for all systems that were studied, including the one-component system. Figure 11a shows the two-component system at $T^* = 0.3$ ($T = 1.2T_m$) in the melt. A large and well-formed percolation string of five- and seven-sided polygons is clearly visible. There are other broken strings of liquid-like polygon dipoles also visible, which can connect up to the main chain and thereby replace other segments of that chain by a statistically equivalent chain in other snapshots in time. Figure 11b shows the polygons in the simulation cell at $T^* = 0.2$ ($T = 0.8T_m$), which is an undercooled melt between T_m and T_g . A percolating string of liquid-like material is still evident in the figure. It is well to recall that the simulation involving step-wise lowering of the temperature followed by only 10^3 time steps for equilibration of the structure constitutes a quenching rate of about 10^{11} K s^{-1} . In figure 11c, the system is shown at $T^* = 0.14$ ($= 0.78T_g = 0.56T_m$), where many associated dipoles of five- and seven-sided polygons are visible, but the string-like percolation of these through the simulation cell is now broken. Thus, the material has apparently regained its solid-like behaviour. Finally, in figure 11d, at a temperature of $T^* = 0.06$ ($= 0.33T_g = 0.24T_m$), some five- and seven-sided polygon dipoles still exist, but are now isolated in a background of short range ordered material.

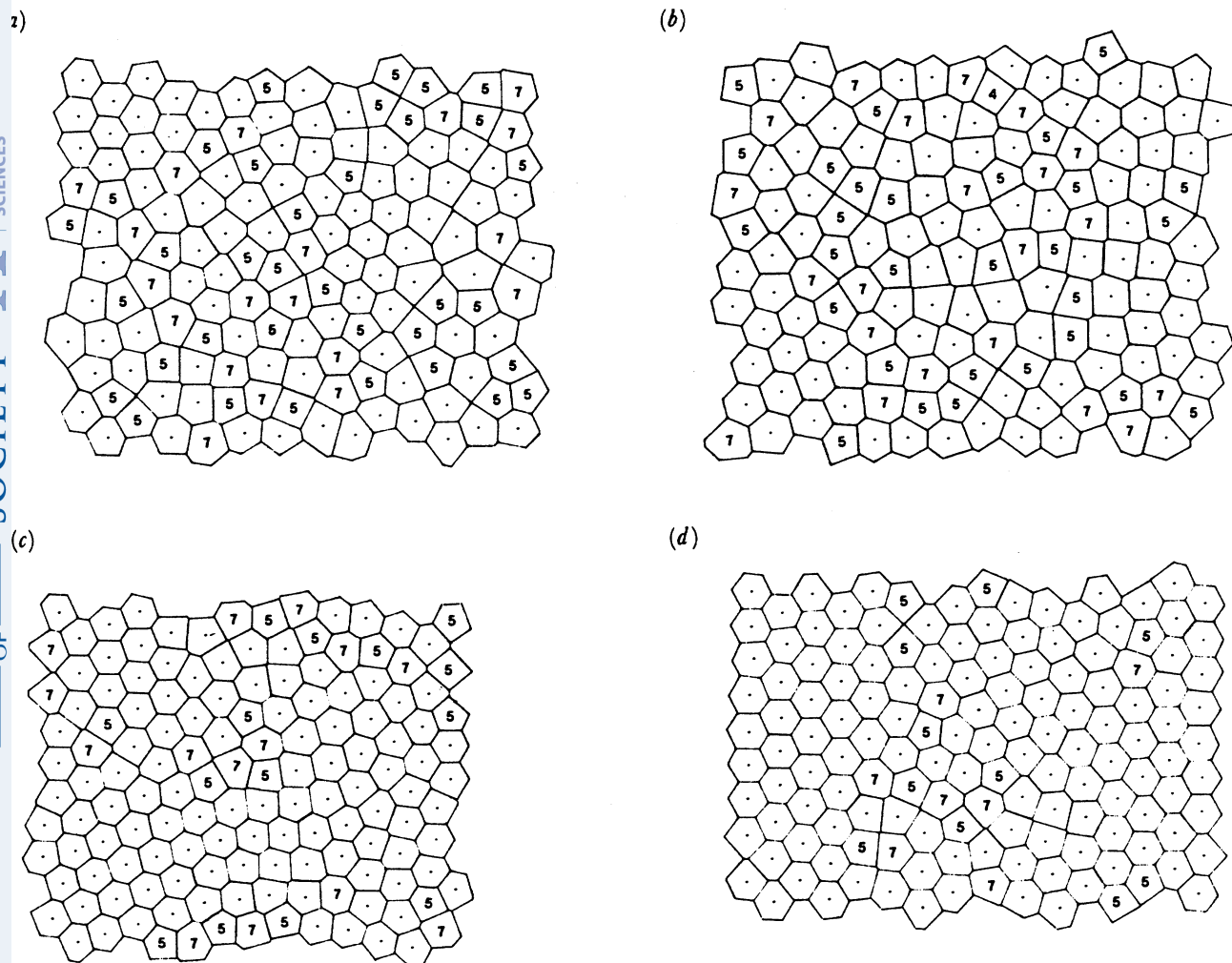


FIGURE 11. Temperature dependent alterations in the structure of the two-component material upon quenching at an external pressure of $p^* = 1.0$: (a) $T^* = 0.3$ ($T = T_m$); (b) $T^* = 0.2$ ($T_g < T < T_m$); (c) $T^* = 0.14$ ($T < T_g$); (d) $T^* = 0.06$.

4. DISCUSSION

4.1. Melting and glass transition

Our simulation of two-dimensional hexagonal crystals has shown that in the first-order transition of melting, the system does not expand by spreading the disorder quasi-uniformly, but rather acquires well-defined imperfections having excess free volume. The principal ingredient of these imperfections are structural dipoles of associated Voronoi polygons of five and seven sides.

When the system is molten so that solid-like behaviour is replaced by liquid-like behaviour, a critical concentration of the material must acquire a radically lowered level of cohesion. This occurs through the establishment of a percolation condition, in which strings of five- and seven-sided polygon dipoles pervade the entire simulation cell in one or more directions and hence, through the periodic boundary conditions, reach over the entire infinite two-dimensional space.

Clearly, for this picture to have meaning, the 5–7 dipole must have elemental liquid-like properties. That this is so can be gleaned from figure 12, which gives the average areas of the Voronoi polygons in units of r_0^2 , of five, six and seven sides, as measured from actual simulation cells at a temperature of $T^* = 0.1$. These relative differences do not change much with temperature up to the melting point. As the figure shows, the average heptagon is about 10% larger than the average hexagon, which on the average is only 3% larger than the average pentagon. Thus, the area of the average dipole of 5–7-sided Voronoi polygons is about 7% larger than a pair of average hexagons. As the average hexagons themselves are about 9% larger at the melting point than the hexagons at 0 K, the average dipole in the melt is about 16% larger than a pair of hexagons in a two-dimensional material at rest at 0 K. This is a substantial increase, which should put the material of the dipole at or beyond the decohesion strain, and give it indeed liquid-like properties. We show in parts II and IV that both the structural relaxations and plastic deformation at low temperatures are strongly influenced by the liquid-like material.

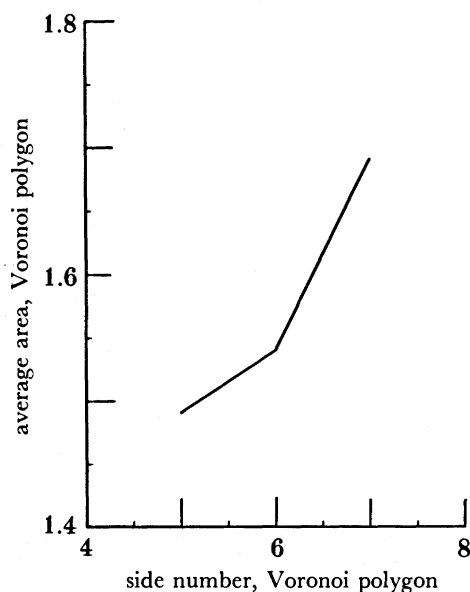


FIGURE 12. Dependence of the average area of the Voronoi polygons on the number of sides of the polygons, in the two-component material at $T^* = 0.1$.

The importance of such liquid-like material having excess free volume that percolates through the structure has been postulated earlier by Cohen & Grest (1979), and by Grest & Cohen (1981) in relation to a glass transition and establishment of overall liquid-like behaviour in bulk. Our simulations are in support of these views.

Our simulation has demonstrated further that glass transition takes place when, in an undercooled melt, the percolation condition established by the strings of liquid-like material is broken. This view is also not entirely new, and although it has been set forth from time to time by others (Meares 1965), it has not been widely favoured.

A related and complementary concept, that of percolation of a property called rigidity, was developed independently at the same time by Phillips (1979, 1981). This was later quantified by Phillips & Thorpe (1985) and formulated as a process known as vector percolation by Thorpe (1983). In this concept, one speaks of random networks consisting of floppy (or spongy)

and rigid regions; isolated rings (polygons) with less than six sides would be rigid, while rings with seven or more sides would be floppy. When rings become part of a network, the rigid rings remain rigid, but a floppy ring now can become rigid if it is next to a rigid ring, and in this manner, rigidity can percolate through the network (Thorpe 1983). Although rigidity percolation was developed for network glasses, and has other applications to gellation, free volume percolation on the other hand is more pertinent to metallic glasses and the establishment of fluidity. We see, nevertheless, a certain analogy between these basic concepts and the percolation of structural dipoles observed in our simulation.

4.2. Topological defects

The association of the five- and seven-sided polygons into a structural dipole with a definite increment of free volume makes it a characteristic unit in the description of the molten state, as well as the glassy state in two-dimensional condensed matter. It is interesting that both in the molten state at the melting temperature, and in the glassy state below T_g , disorder in the material is not spread out uniformly over all volume elements. Rather, a considerable fraction of the material at any instance in time, maintains a reasonable degree of short range order while squeezing the disorder into the borders between the ordered regions, where it is accommodated in the form of the 5–7-sided structural dipoles. Thus, this arrangement must have a lower energy state than the one where atomic disorder is quasi-uniformly spread out over the entire two-dimensional space. Clearly, also, the 5–7-sided dipole cannot be rearranged into a pair of distorted six-sided average polygons, because that would have a lower free volume, which apparently is prevented from becoming that by the constraint imposed by the surrounding regions of short-range ordered material.

The 5–7-sided dipole of Voronoi polygons combining a region of excess dilatation and one with a smaller compaction would appear to have the features of an edge dislocation, and could readily be interpreted as such. To explore this possibility, we have analysed a number of well-defined lattice defects, such as single vacancies, di-vacancies, tri-vacancies, and tetra-vacancies, edge dislocations, and stacking faults that have been found in Bragg bubble rafts, which have the advantage of being readily available from the literature (Fukushima & Ookawa 1955), and being fully equilibrated through an inter-bubble potential resembling closely certain inter-atomic potentials of metals (Shi & Argon 1982). Figure 13*a–d* show respectively a vacancy, di-vacancy, tri-vacancy, and tetra-vacancy in bubble rafts, together with their Voronoi polygon representations. We see that in each case these vacancy type clusters are delineated by a pair of 5–7-sided dipoles, in which the seven-sided Voronoi polygons face each other, and their interfaces with the five-sided polygons are separated by from one to four intervening polygons. If the vacancies were aligned in a line, such arrangements can indeed be represented by a pair of edge dislocations with their tensile fields facing each other, terminating the rows of 1, 2, 3, and 4 vacancies. Figure 14*a, b* (Fukushima & Ookawa 1955; McClintock & Argon 1966) show two edge dislocations in two bubble rafts with their associated Voronoi polygon descriptions. The dislocation in figure 14*a* is characterized by two adjoining five-sided polygons facing two rather dilated six-sided polygons across a common straight border, while the dislocation in figure 14*b* is represented by a dipole of five- and seven-sided polygons. It is easy to demonstrate that if the dislocation in figure 14*a* is translated slightly toward the left or the right, one five-sided polygon can be transformed into a distorted six-sided polygon, while the large six-sided polygon below the remaining five-sided polygon is transformed into a seven-sided polygon to

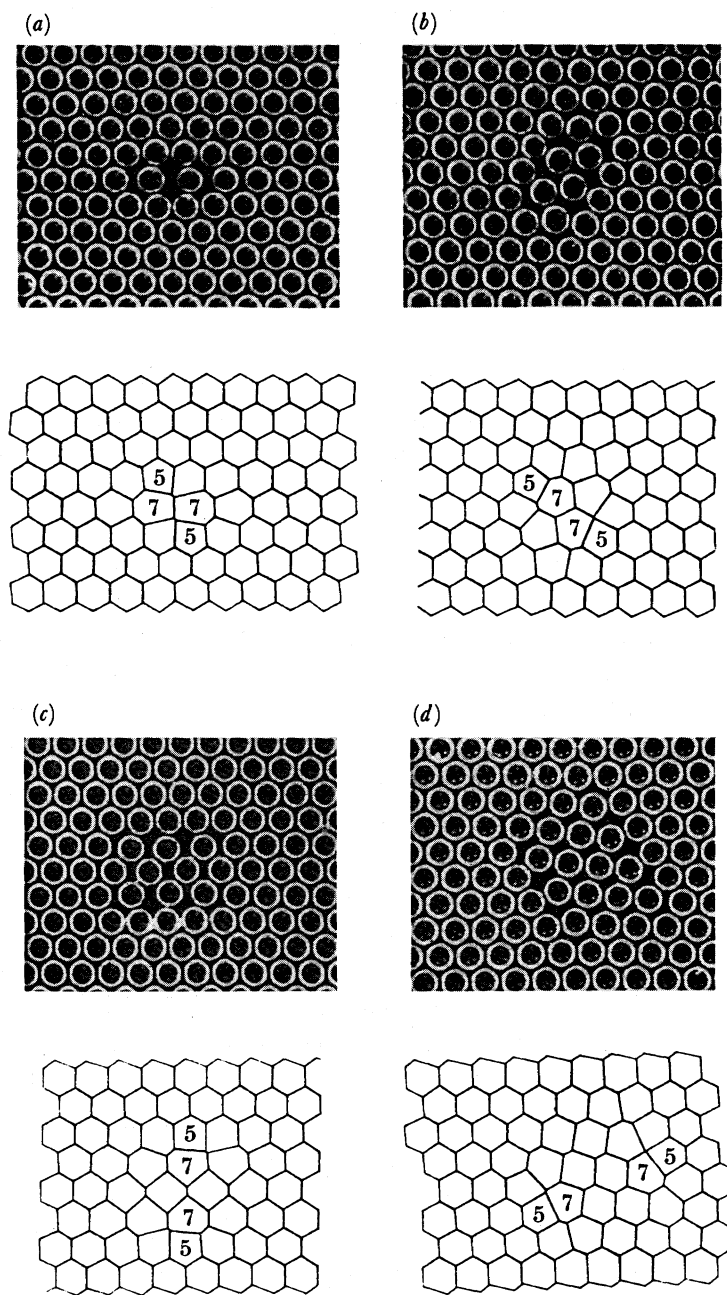


FIGURE 13. Distortions produced in the Voronoi polygon field of a perfect hexagonal bubble raft produced by specific vacancy clusters: (a) vacancy; (b) di-vacancy; (c) tri-vacancy; (d) tetra-vacancy.

look just like the dislocation in figure 14*b*. Figure 15*a, b* shows two stacking faults in a bubble raft (Fukushima & Ookawa 1955). Although the one in figure 15*a* is planar, and is terminated by two partial dislocations, the one in figure 15*b* is complex and distributed over a volume. Thus, in all of these instances, in the description of these crystal defects, edge dislocations play a role. When the dipoles occur in closely spaced pairs with the seven-sided polygons facing each other within one or two atomic positions, the defects are clearly of vacancy character in a perfect crystal environment, and an element of large free volume in an amorphous environment of

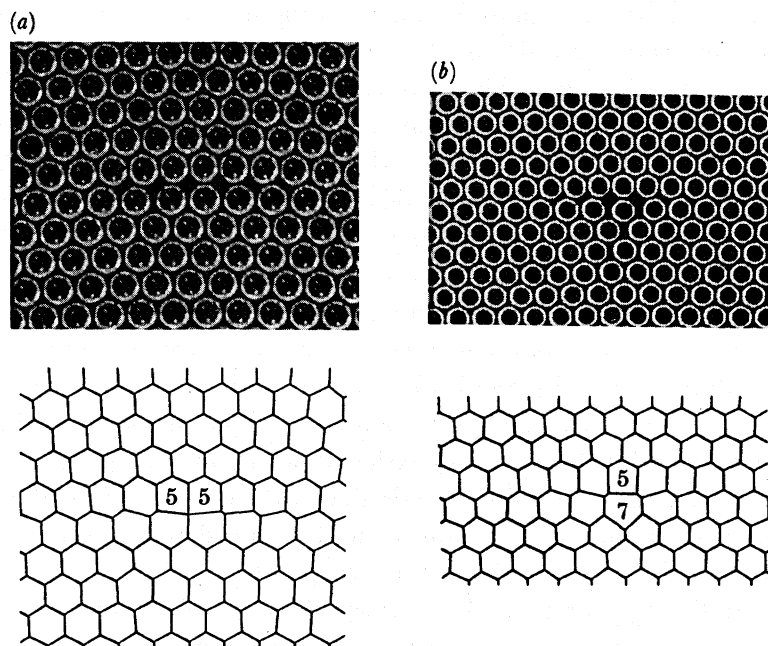


FIGURE 14. Distortions produced in the Voronoi polygon field of a perfect hexagonal bubble raft produced by edge dislocations: (a) edge dislocation in an intermediate position between two low energy configurations; (b) edge dislocation in the low energy configuration.

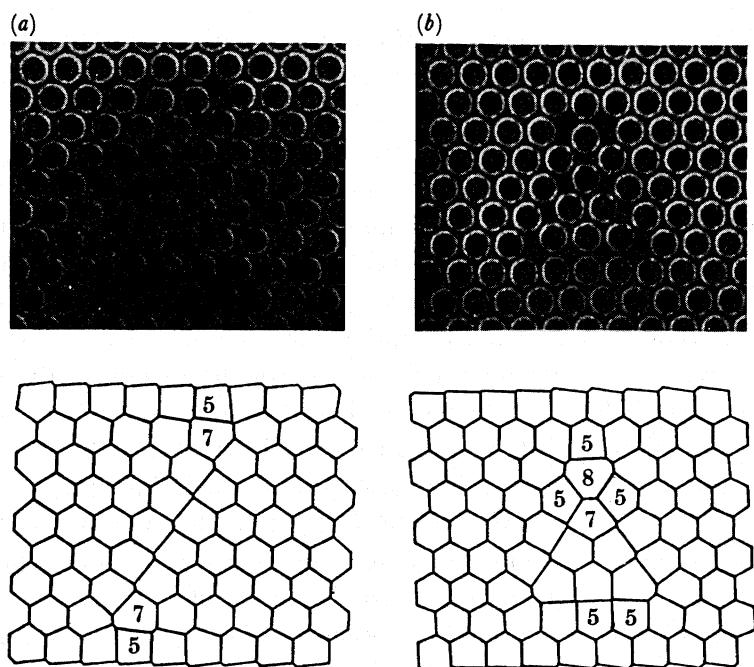


FIGURE 15. (a) A stacking fault with two terminating partials in a perfect hexagonal bubble raft, and (b) a complex two-dimensional fault.

distorted hexagons. Such defect pairs were found in significant concentrations in amorphous Bragg bubble rafts by Argon & Shi (1982) as sets of closely spaced Burgers vector couples.

In the strings of percolating liquid-like material in the melt, subcooled melt, and the rapidly quenched glass, the 5–7-sided dipoles in touching configurations are predominantly ordered in sequences of 5–7–5–7, etc. This indicates that the material of the strings acts in the nature of a high-angle grain boundary with the geometrically necessary dislocations being placed so close as to have their cores penetrate into each other. Thus, as is well known from the study of such high angle boundaries, their dislocation nature should then be completely lost, and their topological contents must be viewed as sets of polyhedra (polygons in two dimensions) (Ashby *et al.* 1978). Hence, the structure of the melt consists of small islands of quasi-crystalline order separated by boundaries of quasi-periodically arranged polygons of 5–7 sides, retaining their topological signatures as stacked edge dislocation cores. As we demonstrate in part II, in the melt, the liquid-like material (boundary material) typically makes up a fraction of about 0.4 of the total. Thus the quasi-crystalline islands that are enclosed within the boundaries are exceedingly small indeed, and are internally somewhat disordered as well. It is interesting to note that the make-up of static amorphous Bragg soap bubble rafts composed of two dissimilar size bubbles and studied by Argon and co-workers (Argon & Kuo 1979; Argon & Shi 1982) had structures very similar to the structure of the two-dimensional mats in the present simulations. In addition, the melting simulation carried out by Fukushima & Ookawa (1955) by mechanically vibrating Bragg soap bubble rafts made up of bubbles of one size have also given melt structures that were remarkably similar to results of our simulation, even though in their case, the amplitudes of vibration were largely normal to the plane of the raft, rather than being in the plane.

The role of dislocations in melting and dislocation theories of melting are not new (for a discussion of some early attempts see Nabarro (1967)). Most of such developments were based on the observation that the energies of arrays of dislocations increase less than linearly, while the configurational entropy increases linearly, resulting in a well-defined discontinuous behaviour at a definite temperature with a critical concentration of dislocations. Among the more modern studies is the computer simulation of Koizumi & Ninomiya (1980), who have attained a glassy state by progressively increasing the density of screw dislocations of all types in a diamond cubic or face centred cubic (f.c.c.) lattice, followed by relaxing the positions of the atoms on the basis of an atomic pair potential. These authors have reported reaching RDFs that were quite similar to those obtained for amorphous metals, with total densities of dislocations in the range of 10^{14} – 10^{15} lines cm^{-2} , i.e. where the dislocation cores occupy nearly the entire volume. While these authors do not report the final structure in more detail, they make the observation that core structures of dislocations remain intact in the final amorphous material. Similar results were reported also by Kotze & Kuhlmann-Wilsdorf (1971).

The above observations bring to the fore long-standing controversies on the structure of the amorphous state and more particularly the role generalized dislocations could play in the inelastic deformation of such media (for a discussion, see Argon 1981). We demonstrate in our associated simulation on plastic flow (see part IV), in the two-dimensional amorphous media, that whereas most of the inelastic deformation is assignable to local shear transformations occurring inside the liquid-like materials of the boundaries, a small component of strain also comes from mobility of dislocations in the quasi-crystalline domains encapsulated by the liquid-like material. As can be expected, this latter component of deformation assumes greater

importance in well-relaxed glasses, with a high crystallinity where well-formed and isolated dislocations are readily recognizable inside the ordered region, as can be seen in figure 16 at sites A and B.

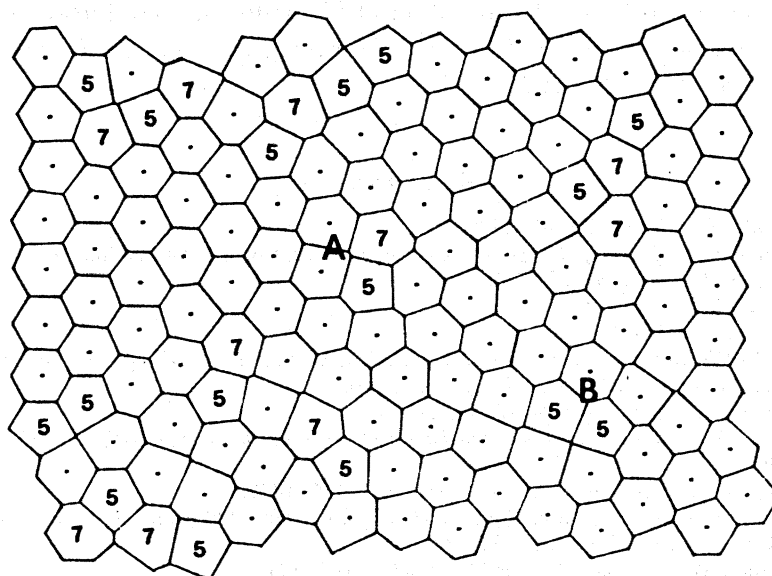


FIGURE 16. Some isolated edge dislocations (5-7-sided dipoles) in a well-relaxed two-component glass.

4.3. Generalization of results

Although our observations of the thermo-physical properties of a two-dimensional idealized medium and its phase transitions qualitatively reproduce nearly all features that three-dimensional media exhibit, the results should be taken with some degree of caution. First, constraining the atoms to exist only in two-dimensional mats prevents the local configurations from reaching lower energy forms by moving in all three directions. This invariably incorporates a higher free volume into the structure than is likely to be the case in three-dimensions, and may artificially enrich the liquid-like material. This must be one reason why the melting points found in our simulations were significantly lower and the coefficients of expansion higher than expected in comparison with three-dimensional material.

In three dimensions, we visualize the string-shaped percolation of the liquid-like material to take the form of tortuous boundaries pervading space and encapsulating small quasi-crystalline domains. The projection into three dimensions of closely spaced 5-7-sided polygons representing liquid-like material require replacing them with pairs of polyhedra of the type that make up high-angle grain boundaries (Ashby *et al.* 1978). In well-relaxed glassy materials with a high degree of crystallinity, dislocation lines should exist, and can be of not only edge type, but also screw and of mixed nature. It is unlikely, however, that they play any more important role in the description of the structure or its phase transitions.

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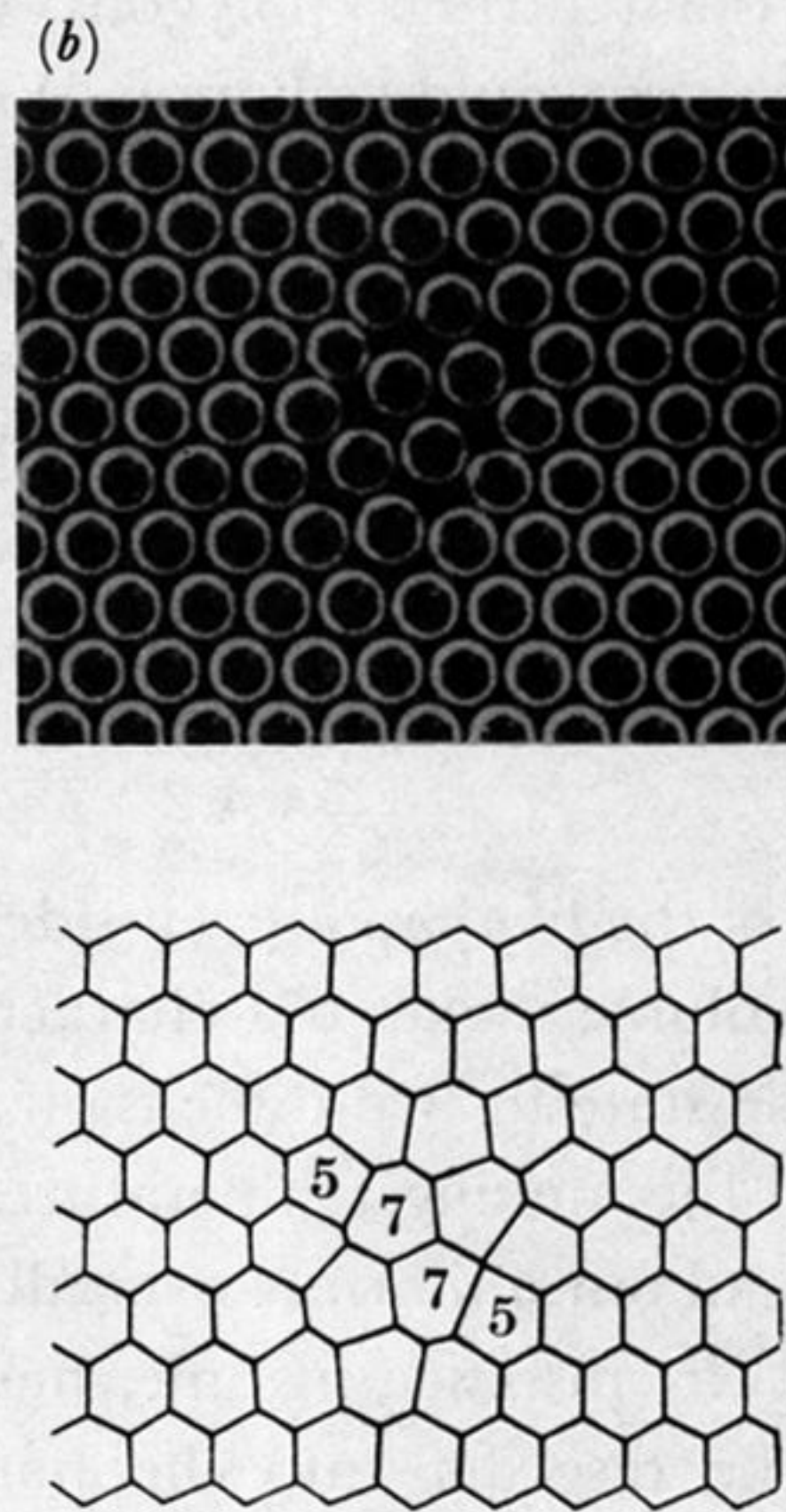
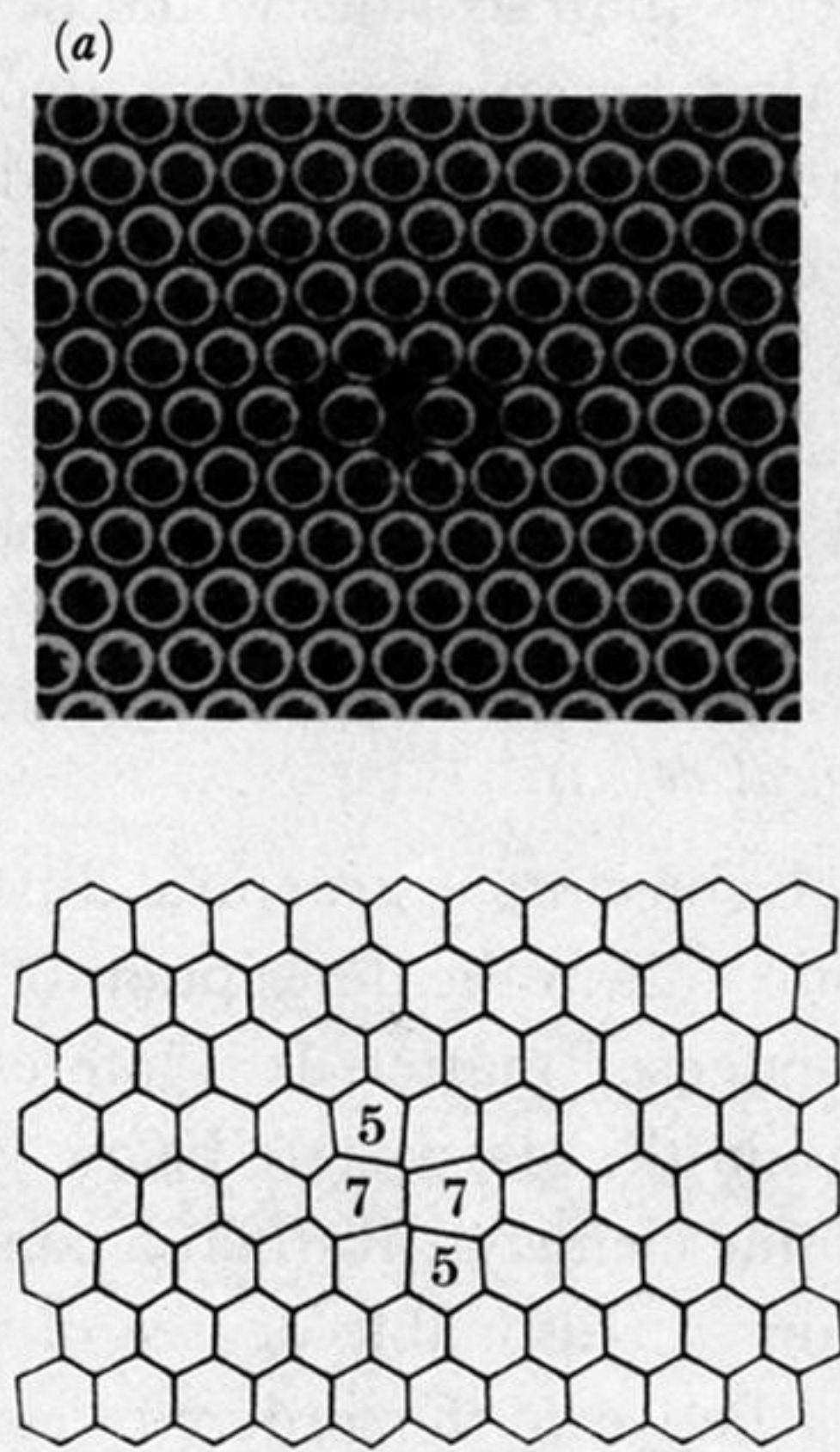
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Note added in proof (22 September 1989). In §3.1 we indicated that our simulated melting points were too low in comparison with those of either a noble gas solid, such as argon, or a simple inter-metallic compound such as Cu–Zr, for which the chosen pair potentials were intended. Here it should be noted that a direct comparison with experiment should be made by using a simulation of thermodynamic melting. For the latter the preferred simulation should be based either on achieving a stationary condition for an interface between a crystal and a melt, or on a direct calculation of the Gibbs free energy of the solid and the liquid phases. For a comparative study of melting in the presence of a free surface or a movable interface (thermodynamic melting), and confined melting in the absence of any surface or interface (mechanical instability) see Phillpot *et al.* (1989).

Reference

Phillpot, S. R., Lutsko, J. F., Wolf, D. & Yip, S. 1989 *Phys. Rev. B* **40**, 2831.



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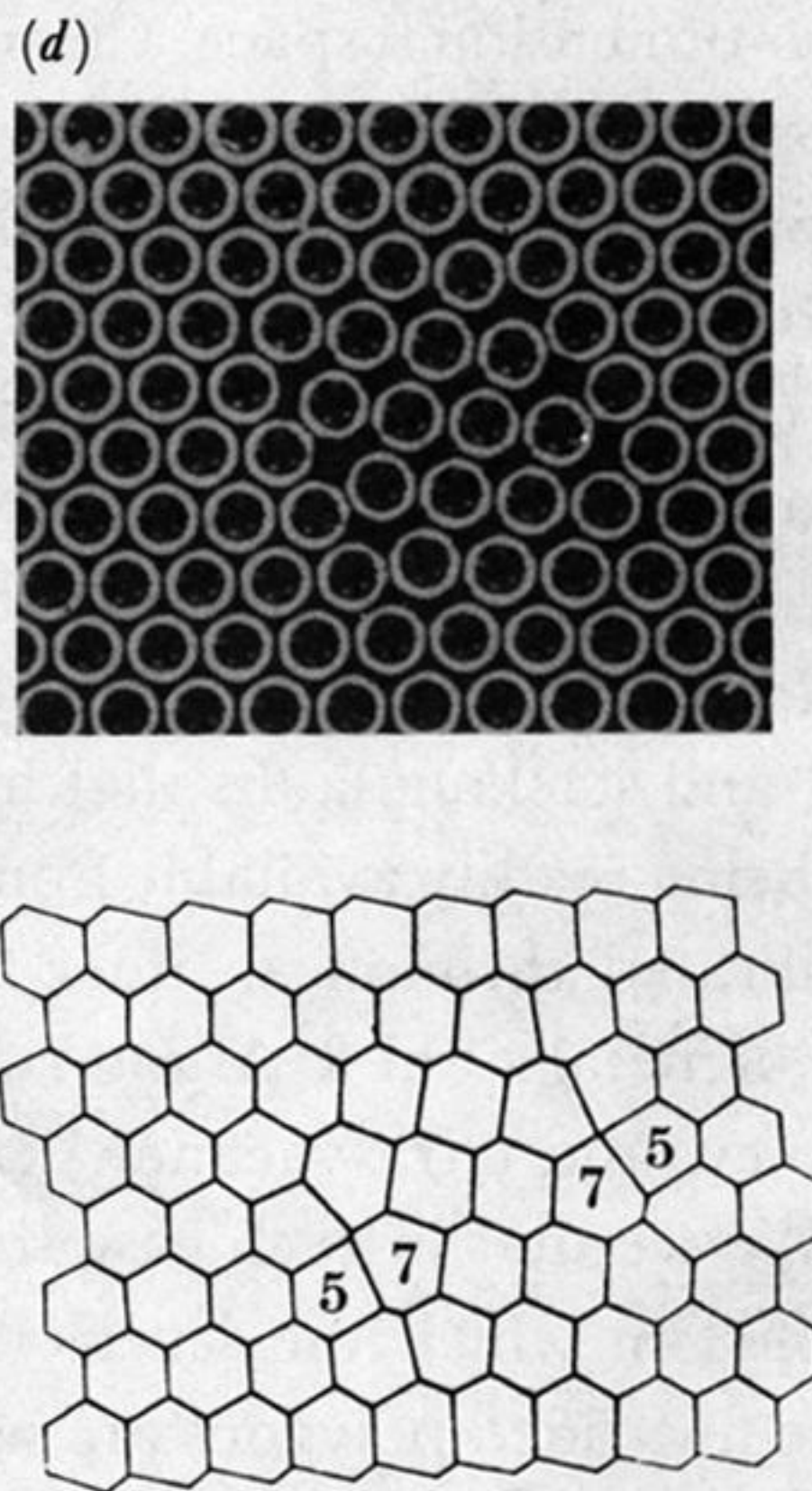
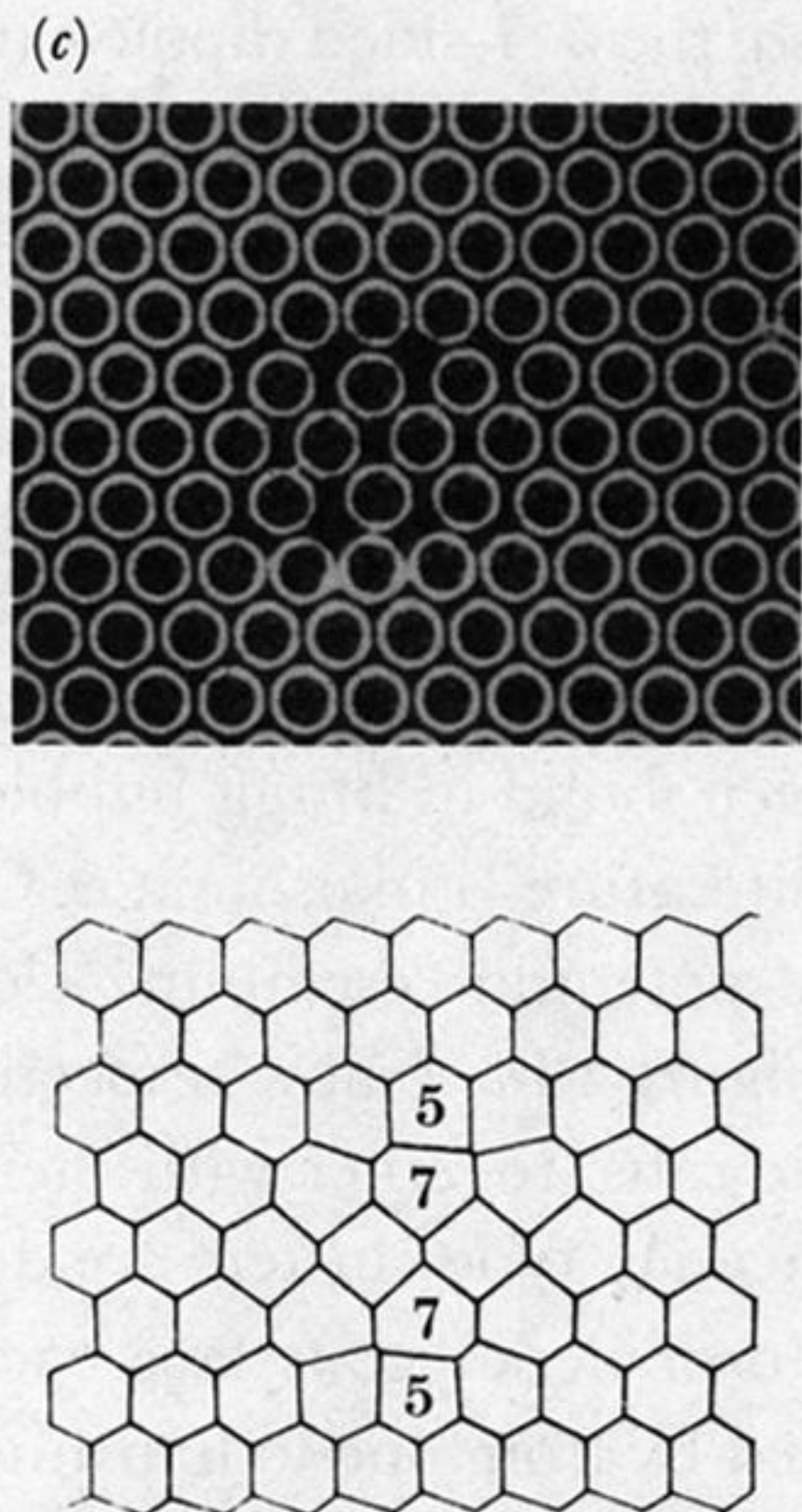


FIGURE 13. Distortions produced in the Voronoi polygon field of a perfect hexagonal bubble raft produced by specific vacancy clusters: (a) vacancy; (b) di-vacancy; (c) tri-vacancy; (d) tetra-vacancy.

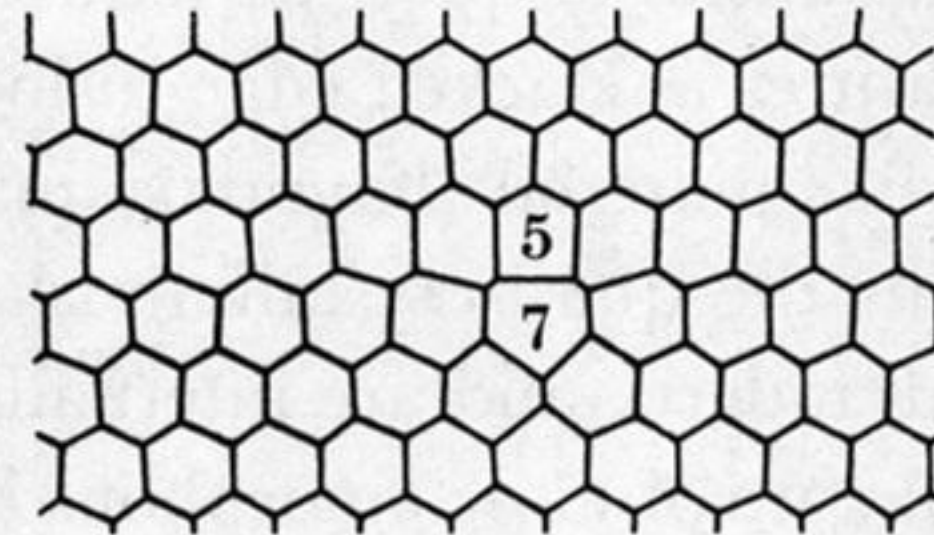
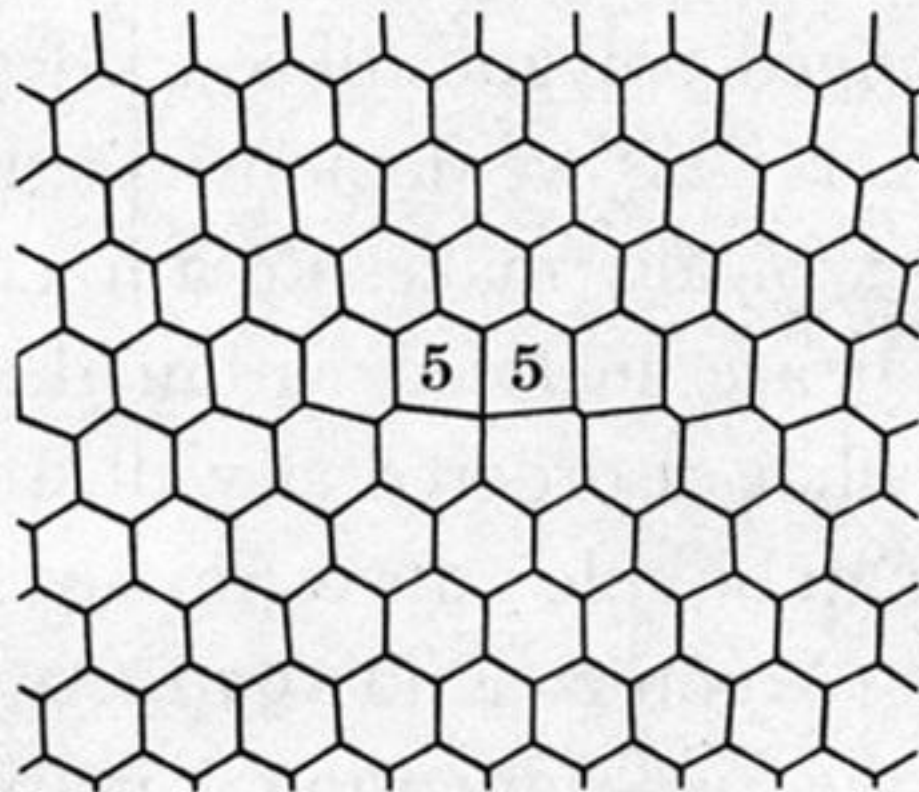
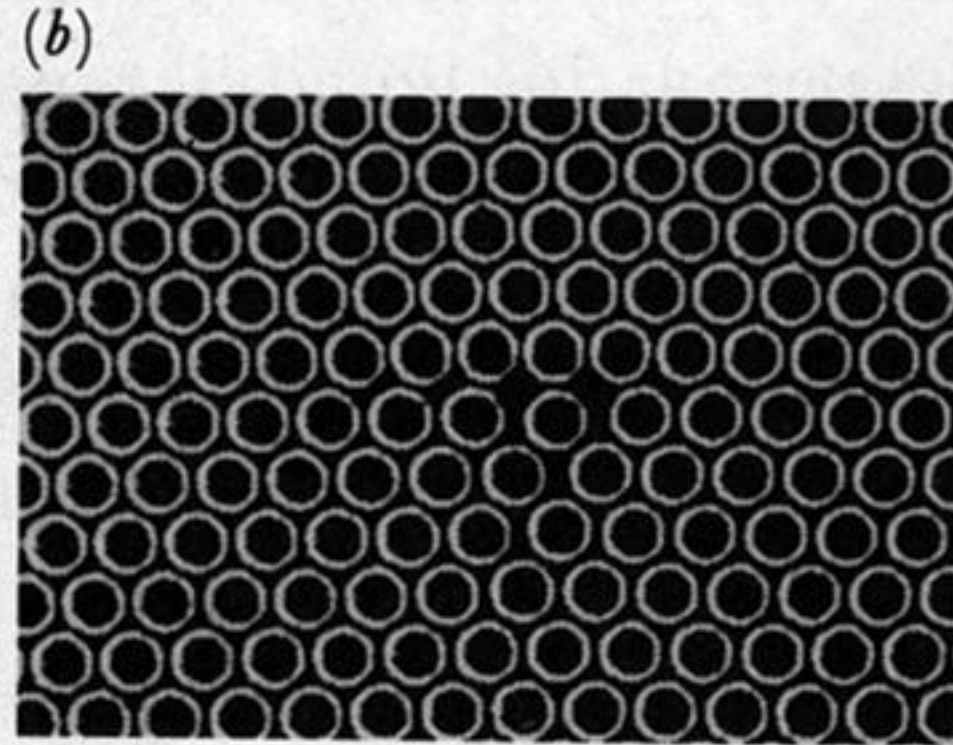
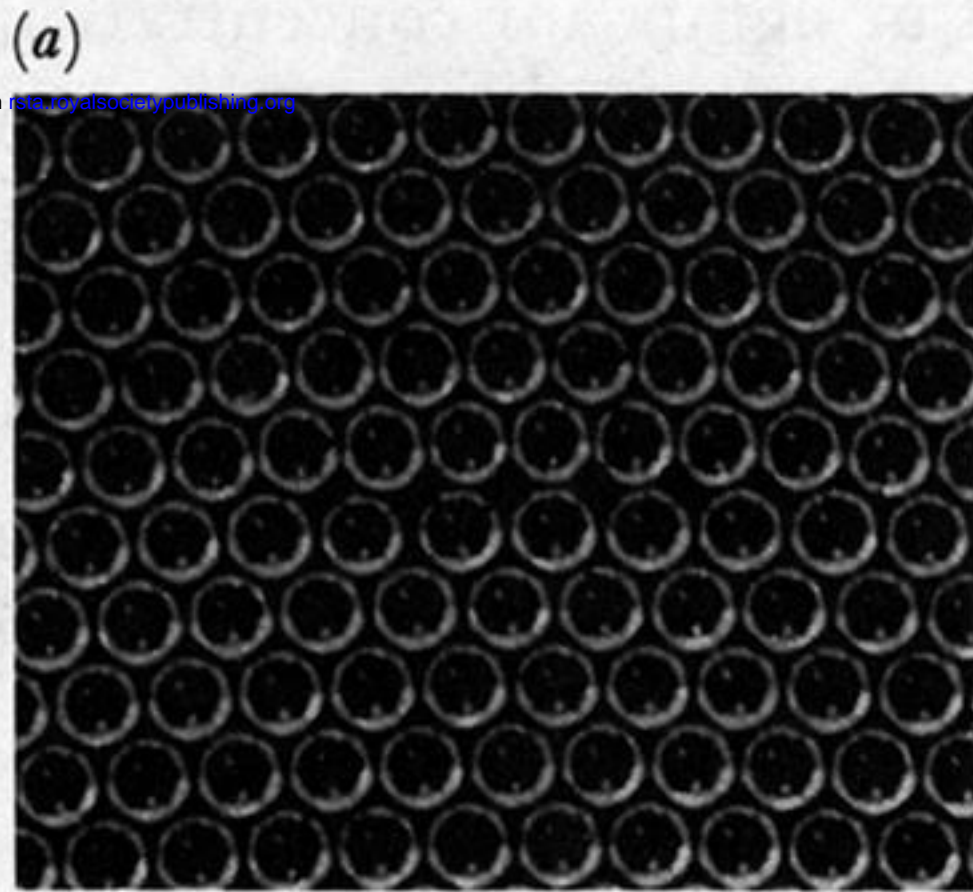
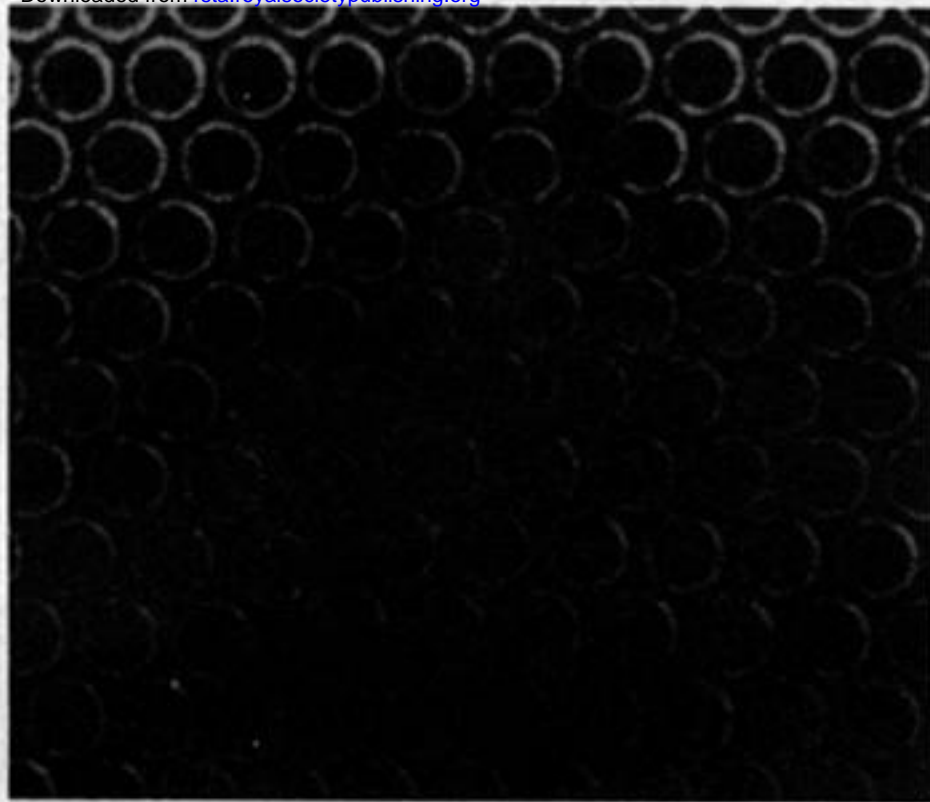


FIGURE 14. Distortions produced in the Voronoi polygon field of a perfect hexagonal bubble raft produced by edge dislocations: (a) edge dislocation in an intermediate position between two low energy configurations; (b) edge dislocation in the low energy configuration.

(a)

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(b)

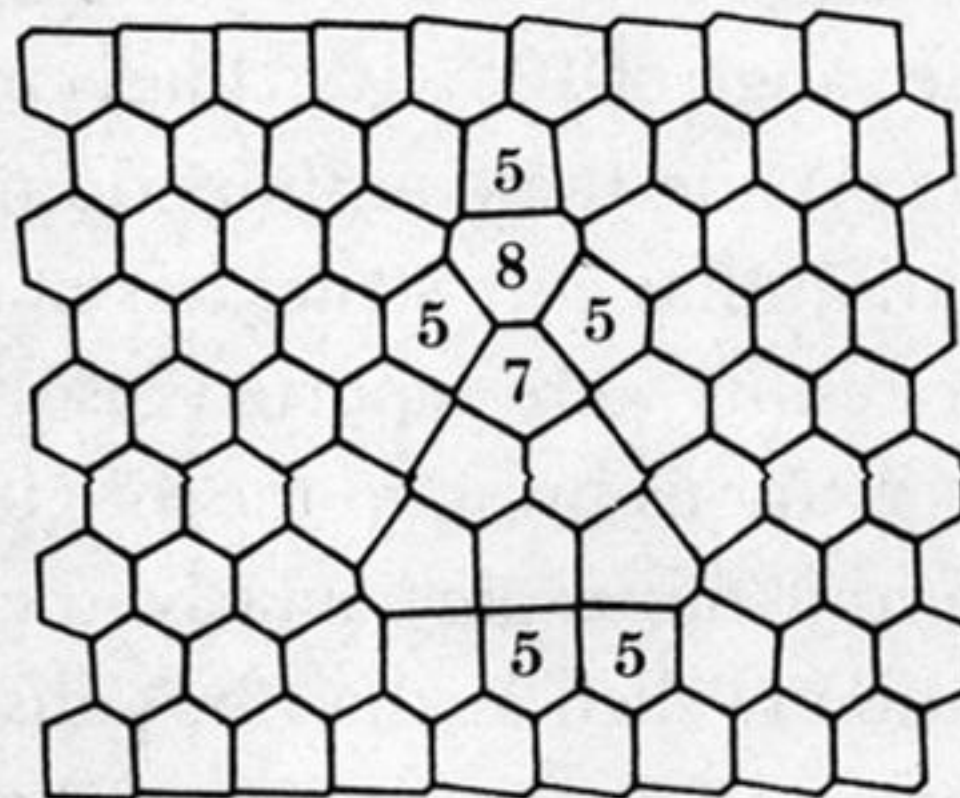
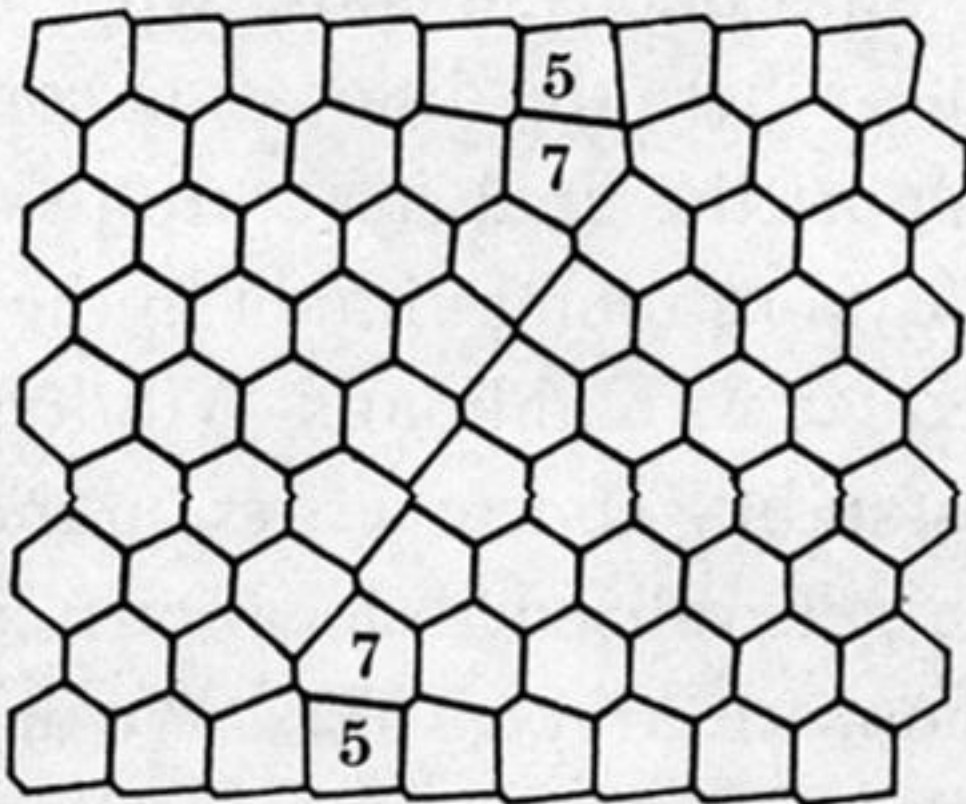
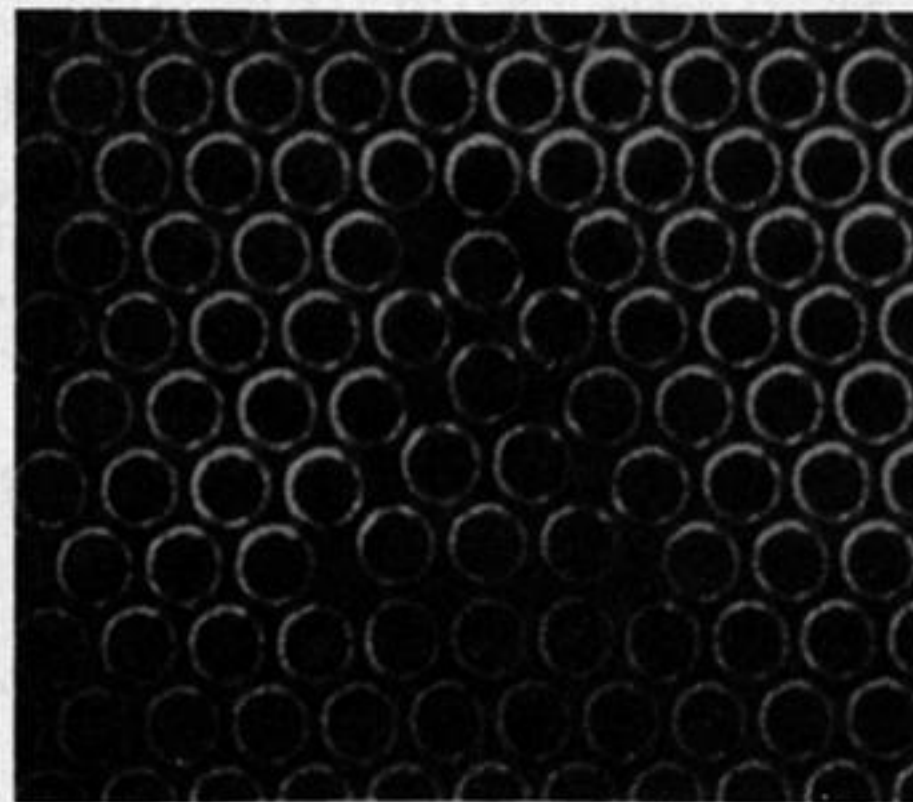


FIGURE 15. (a) A stacking fault with two terminating partials in a perfect hexagonal bubble raft, and (b) a complex two-dimensional fault.